A15273W1

Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

SUPERSYMMETRY AND SUPERGRAVITY

Trinity Term 2018

FRIDAY, 20TH APRIL 2018, 2.30pm to 5.30pm

You should submit answers to all three questions.

You must start a new booklet for each question which you attempt. Indicate on the front sheet the numbers of the questions attempted. A booklet with the front sheet completed must be handed in even if no question has been attempted.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do not turn this page until you are told that you may do so

1. (a) [5 marks] Show that

$$\begin{aligned} P_\mu &= -i\partial_\mu \\ Q_\alpha &= -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\,\bar\theta^{\dot\beta}\partial_\mu \\ \bar Q_{\dot\alpha} &= i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\gamma\,(\sigma^\mu)_{\gamma\dot\alpha}\,\partial_\mu \end{aligned}$$

satisfy the $\mathcal{N}=1$ supersymmetry algebra.

(b) [7 marks] Write down the form of a general scalar superfield. How can this be restricted to a chiral superfield, and what is the resulting form of a chiral superfield? Define any terms you introduce.

(c) [5 marks] Explain the physical meaning of the different components of a chiral superfield. What is the general form of the action - both in superspace and in components - for a single uncharged chiral superfield?

(d) [4 marks] A chiral superfield Φ has the property that $\phi \to \phi + i$ is an exact symmetry of the theory (where ϕ is the scalar component of the superfield). How does this symmetry restrict the effective action?

(e) [4 marks] Construct an example of a non-trivial theory (i.e. one with non-vanishing superpotential) that contains this symmetry and possesses a supersymmetric vacuum solution.

2. (a) [4 marks] State without proof the Coleman-Mandula theorem and explain how supersymmetry evades it.

(b) [7 marks] Construct the explicit map from $SL(2,\mathbb{C})$ to $SO(3,1)$. Why is this map not an isomorphism?

(c) [7 marks] State the supersymmetry algebra and explain what an R-symmetry is. Give an example of a continuous R-symmetry and show explicitly how members of a supersymmetry multiplet can have different R-charges.

(d) [4 marks] Define the discrete R-parity present in the MSSM and give two motivations for introducing it.

(e) [3 marks] For $\mathbb{N}=4$ supersymmetry in 4 dimensions, what is the largest possible R-symmetry group?

3. (a) [15 marks] Explain how to construct theories of supersymmetric Abelian gauge theories. Your answer should include

(i) a definition of a vector superfield
(ii) the implementation of gauge transformations in supersymmetric theories
(iii) the utility of Wess-Zumino gauge
(iv) the field strength superfield and a proof that it is gauge-invariant
(v) the form of the action in both superspace and component form

Now consider a theory of a classical supersymmetric U(1) gauge theory coupled to a single charged chiral multiplet.

(b) [3 marks] What can you say about the form of the superpotential?

(c) [5 marks] Suppose there is a non-zero Fayet-Iliopoulos term ξ. Describe the dynamics that this gives rise to (consider both signs of ξ).

(d) [2 marks] Why are Fayet-Iliopoulos terms only allowed in Abelian gauge theories?