A15273W1

Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

SUPERSYMMETRY AND SUPERGRAVITY

Trinity Term 2016

FRIDAY, 22 APRIL 2016, 14.30 to 17.30

You should submit answers to three of the four questions.

You must start a new booklet for each question which you attempt. Indicate on the front sheet the numbers of the questions attempted. A booklet with the front sheet completed must be handed in even if no question has been attempted.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do not turn this page until you are told that you may do so

1. (a) [4 marks] Write down the supersymmetry algebra in four dimensions, and explain the differences between $N = 1$ supersymmetry and extended supersymmetry.

   (b) [6 marks] Prove that, with one exception, there are equal numbers of bosons and fermions in any finite-dimensional representation of the supersymmetry algebra. Briefly describe that exception.

   (c) [10 marks] For the case of massless particles, explain how to construct representations of the supersymmetry algebra. Illustrate your answer by describing the construction of the chiral and vector multiplets for $N = 1$ supersymmetry, the vector multiplet for $N = 4$ supersymmetry, and the gravity multiplet for $N = 8$ supersymmetry.

   (d) [2 marks] Why are the only chiral supersymmetric theories those with $N = 1$ supersymmetry?

   (e) [3 marks] Determine the particle content of $N = 3$ supersymmetry multiplets without gravity, and comment on the relation, if any, to $N = 4$ multiplets.

2. (a) [6 marks] Explain what is meant by a *chiral superfield*, and write out its expansion in superspace. Your answer should include a description of the field content of the chiral superfield, and also the number of physical degrees of freedom that it contains. You should define any operators you introduce.

   (b) [2 marks] Explain why the product of any number of chiral superfields is also a chiral superfield.

   (c) [10 marks] Consider the theory of a single uncharged chiral superfield Φ.

      (i) Write down the general form of the action in superspace, for a Kähler potential $K(\Phi, \Phi^\dagger)$ and a superpotential $W(\Phi)$.

      (ii) In the case of a canonical Kähler potential $K = \Phi\Phi^\dagger$ and generic superpotential $W(\Phi)$, carry out the integration over superspace and determine the form of the Lagrangian in as much detail as possible.

   (d) [4 marks] Now suppose that $W = \Lambda^3 \exp\left(-(\Phi/M)\right)$, where Λ and M are constants of mass dimension one. You may regard K as canonical.

      (i) Identify a flat direction in the potential.

      (ii) Does this theory have any supersymmetric solutions?

      (iii) What will happen to the field Φ in this theory?

   (e) [3 marks] Now consider the case $W = \Lambda_1^3 \exp\left(-(\Phi/M_1)\right) + \Lambda_2^3 \exp\left(-(\Phi/M_2)\right)$, where $\Lambda_1, \Lambda_2, M_1$ and M_2 are all constants of mass dimension one. You may again regard K as canonical. Does this theory have a flat direction? Does it have a supersymmetric solution?

3. (a) [6 marks] Describe *three* phenomenological reasons for searching for supersymmetry at the TeV scale.

   (b) [5 marks] Write down all the superfields, together with their $SU(3)_c \times SU(2)_L \times U(1)_Y$ charges, necessary to construct the Minimal Supersymmetric Standard Model (MSSM).

   (c) [4 marks] Explain why the MSSM requires two Higgs doublets whereas the Standard Model only has one (provide two reasons).

   (d) [2 marks] How many real physical scalar Higgs bosons are there in the MSSM? Explain your reasoning.

   (e) [5 marks] Write down all gauge-invariant renormalisable or super-renormalisable operators that one can construct using the MSSM superfields. Assuming that all operators are allowed and with $\mathcal{O}(1)$ coefficients, estimate the proton lifetime with TeV-scale superpartners. Explain how the result motivates the idea of *R-parity*.

(f) [3 marks] Construct a gauge-invariant non-renormalisable term in the superpotential involving only Higgs and lepton superfields. What physical phenomenon does this term lead to after electroweak symmetry breaking?

4. (a) [1 mark] How is a *vector superfield* defined?

(b) [5 marks] Write down the general form in superspace of the action for an $N = 1$ supersymmetric gauge theory coupled to charged chiral superfields (you may assume that the superpotential vanishes and that the chiral superfiels have canonical kinetic terms, and may also restrict to the case of an Abelian gauge theory). Define all expressions you introduce.

(c) [4 marks] Explain the *general* structure of the resulting action (factors of 2 or i are not important), including the form of the D-term potential (there is no need to explicitly derive the action from the superspace expression in the previous part).

(d) [2 marks] What is an *anomaly* in the context of quantum field theory? What is the difference between global and gauge anomalies?

(e) [5 marks] Define the *Konishi anomaly*, including an explanation of the differences between 1PI and Wilsonian effective actions.

(f) [8 marks] *Outline* the computation of the Konishi anomaly. While a full calculation is not expected, you should demonstrate an understanding of the overall structure of the calculation, and the different parts that feed into the overall result.