Final Honour School of Mathematical and Theoretical Physics: Part C

Master of Science in Mathematical and Theoretical Physics

STELLAR ASTROPHYSICS

TAKE-HOME PAPER

TRINITY TERM 2016
SATURDAY, 18 JUNE 2016, 2:30pm to TUESDAY, 21 JUNE 2016, 2:30pm

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Answer **all** *of the questions in Section A and Section B and* **one** *of the two essay questions from Section C. Answer booklets are provided for you to use but you may type your answers if you wish. Typed answers should be printed single-sided and the pages securely fastened together.*

You may refer to books and other sources when completing the exam but should not discuss the exam with anyone else.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Section A: Advanced Stellar Astrophysics

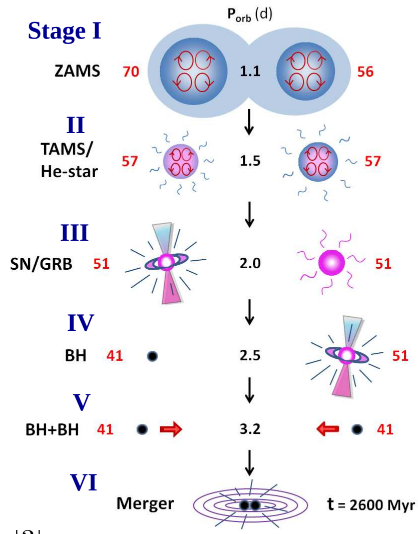
Please answer **all questions** *in this section.*

1. Briefly explain the concept of a *gravitational wave* and how gravitational-wave detectors such as Advanced LIGO (aLIGO) can detect them. Discuss some of the main sources of gravitational waves that can potentially be detected. [8]

The mergers of relatively massive stellar-mass black holes (BHs) are one of the prime sources for aLIGO detections. The figure to the right sketches the evolution of a massive binary system that produces such a black-hole merger. In order for the systems to merge in less than a Hubble time, the orbital separation of the BH binary after the formation of the BH+BH system (stage V in the figure) has to be less than

$$a_{\rm max}^{\rm merger} \simeq 50\,R_\odot \left(\frac{M_{\rm BH}}{50\,M_\odot}\right)^{0.75},$$

assuming an equal-mass system where $M_{\rm BH}$ is the mass of each black hole.
Explain why stellar-wind mass loss in the early evolutionary phases tends to widen the system and why this depends on the metallicity of the stars? [3]



The change of separation a due to stellar-wind mass loss is given by the differential equation $\mathrm{d}a/\mathrm{d}M = -a/M$, where M is the total mass of the binary. The minimum initial separation $a_{\rm min}^0$ (at which two equal-mass stars fill their Roche lobes on the zero-age main sequence) is approximately given

$$a_{\rm min}^0 \simeq 2\,R_\odot \left(\frac{Z}{Z_\odot}\right)^{0.4} \left(\frac{M^0}{M_\odot}\right),$$

where $Z/Z_\odot$ is the metallicity scaled to the solar metallicity and M^0 is the initial mass of each star. Assuming that the initial masses of the two stars are the same and ignoring any mass loss in the two supernovae (stages III and IV in the figure), determine the minimum separation of the BH binary at the beginning of stage V as a function of M^0, $Z/Z_\odot$ and $M_{\rm BH}$. [3]

The table below gives the final BH masses (in units of $M_\odot$) for different metallicities, assuming equal-mass binaries (the columns give the BH masses for the different initial masses, the rows for the different metallicities scaled to $Z_\odot$).

	$M^0 = 30\,M_\odot$	$100\,M_\odot$
$Z_\odot/4$	26	60
$Z_\odot/10$	28	70
$Z_\odot/20$	29	80

Determine the minimum separations for these systems (at the beginning of stage V) and plot them as a function of the initial mass M^0 in a diagram that also shows $a_{\rm max}^{\rm merger}$ for each metallicity. [6]

Discuss the implications of these results (in particular the metallicity dependence) for detecting such mergers with aLIGO. [5]

2. What is a *core-collapse supernova*? Briefly describe the main phases and the final possible fates of a core-collapse supernova and discuss the main theoretical uncertainties in our understanding of these events. [8]

Consider an iron core of mass $M_{\rm Fe}$ at the end of the evolution of a massive star. Taking the density of the core to be constant (with $\rho = 3 \times 10^{10}\,{\rm kg\,m^{-3}}$), show that the gravitational potential energy of such a core is given by

$$E_{\rm grav} \simeq -6 \times 10^{43} \left(\frac{M_{\rm Fe}}{M_\odot} \right)^{5/3} \ {\rm J}.$$

[6]

Now consider that the innermost $M_{\rm c}$ ($< M_{\rm Fe}$) of the iron core collapses to form a compact object, while the outermost part $\Delta M = M_{\rm Fe} - M_{\rm c}$ is ejected in a supernova explosion. Taking $M_{\rm c}$ to be $1.4\,M_\odot$ in all cases, calculate the energy that is required to eject the rest of the core for $\Delta M = 10^{-3}$, 0.1 and $1\,M_\odot$, also taking into account that the supernova shock will have to photo-dissociate any remaining iron which requires about 10^{44} J for each $0.1\,M_\odot$ of iron. [6]

Comparing this to the typical observed energy of a core-collapse supernova ($\sim 10^{44}$ J), speculate what this might tell you about the possible fates of stars with different iron-core masses and any possible associated supernova kicks. [5]

Section B: Basic Accretion Disc Physics: the Shakura-Sunyaev Disc

Please answer **all questions** *in this section.*

3. In the lectures, expressions were derived for the midplane pressure, temperature, and density of a Shakura-Sunyaev (SS) disc, as well as for the thickness, H, all as functions of the radial distance r (see the expression in the handout or eq. (5.49) in Accretion Power in Astrophysics; you may set $f = 1$ in these equations.)

Take the inner and outer edge of the accretion disk to be located at $r_0 = 10^5\,\mathrm{m}$ and $r_{\mathrm{max}} = 10^9\,\mathrm{m}$, respectively, and assume an accretion rate of $\dot{M} = 10^{15}\,\mathrm{kg\,s^{-1}}$. A plausible value to use for the α parameter is 0.1.

Determine the amount of mass stored in the accretion disc and estimate the timescale for "filling" the disc if it were initially empty. [10]

4. Use the expressions for $\rho(r)$ and $H(r)$ from the previous problem to compute an expression for v_r, the radial in-spiral speed of the disc material. Show that for all choices of parameters α and $\dot{M}$, the radial speed $v_r \ll v_{\mathrm{Kepler}}$, as long as one considers radial distances significantly greater than r_0. [5]

5. Write out an integral expression for L_ν of an SS accretion disc, where L_ν is the spectral luminosity (power per unit frequency interval). Treat each annulus in the disc as a black body of temperature $T_{\mathrm{eff}}(r)$ as defined in the lectures.
[Do not try to integrate the expression as it cannot be done analytically.]

For reference, the Planck function is:

$$P(\nu) = \frac{2\pi h\nu^3 c^{-2}}{[e^{(h\nu/kT)} - 1]}.$$

An approximate expression for L_ν can be obtained by

- approximating the Planck function by

$$P(\nu) = 2\pi h\nu^3 c^{-2} e^{-h\nu/kT}$$

,

- setting $f = (1 - \sqrt{r_0/r})^{1/4}$ in the expression for $T(r)$ equal to 1,
- carrying out the integration from $r = 0$ to $r = \infty$.

Show that

$$L_\nu \propto \nu^{1/3}.$$

[10]

Section C: Advanced Topics

Write a short essay on **one** *of the two topics below.*

Essay A: The Magnetorotational Instability

Write an essay on the magnetorotational instability (Balbus-Hawley instability). Specifically, discuss (a) the physical motivation (the viscosity problem), (b) how the instability works, including some simple estimates of its strength (growth rate), (c) applications to accretion discs, and (d) possible limitations of the theory. [25]

Essay B: Super-Eddington Accretion

Write an essay on the possibility of super-Eddington accretion in X-ray binaries in the context of ultraluminous X-ray binaries (ULXs). Specifically, discuss (a) the controversy concerning ULXs, (b) the assumptions behind the classical derivation of the Eddington limit and their limitations, (c) examples of systems accreting above the Eddington limit, and (d) some models/ideas that have been proposed to violate it including a critical assessment of the models. How does this affect the problem of the growth of supermassive black holes found at the centres of galaxies (you may assume that accretion in these systems can potentially exceed the Eddington limit by a factor of $10 - 100$). [25]