# Examiners' Report: Final Honour School of Mathematical and Theoretical Physics Part C and MSc in Mathematical and Theoretical Physics Trinity Term 2019 

October 31, 2019

## Part I

## A. STATISTICS

- Numbers and percentages in each class.

See Table 1.

|  | Numbers |  |  |  | Percentages \% |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 2019 | $(2018)$ | $(2017)$ | $(2016)$ | 2019 | $(2018)$ | $(2017)$ | $(2016)$ |
| Distinction | 40 | $(25)$ | $(31)$ | $(18)$ | 76 | $(60)$ | $(76)$ | $(86)$ |
| Merit | 6 | $(\mathrm{n} / \mathrm{a})$ | $(\mathrm{n} / \mathrm{a})$ | $(\mathrm{n} / \mathrm{a})$ | 11 | $(\mathrm{n} / \mathrm{a})$ | $(\mathrm{n} / \mathrm{a})$ | $(\mathrm{n} / \mathrm{a})$ |
| Pass | 6 | $(17)$ | $(10)$ | $(3)$ | 11 | $(41)$ | $(24)$ | $(14)$ |
| Fail | 1 | $(0)$ | $(0)$ | $(0)$ | 2 | $(0)$ | $(0)$ | $(0)$ |
| Total | 53 | $(42)$ | $(41)$ | $(21)$ | 100 | $(100)$ | $(100)$ | $(100)$ |

Table 1: Numbers and percentages in each class

- Numbers of vivas and effects of vivas on classes of result.

No vivas were held.

- Marking of scripts.

All dissertations and mini-projects were double-marked, after which the two markers consulted in order to agree a mark between them.
All written examinations and take-home exams were single-marked according to carefully checked model solutions and a pre-defined
marking scheme which was closely adhered to. A comprehensive independent checking procedure is also followed.

## B. New examining methods and procedures

The Merit classification was introduced by the University for all students whose Master's course began in October 2019. Therefore, students who commenced their studies on the MSc in Mathematical and Theoretical Physics or the Master of Mathematical and Theoretical Physics (MMathPhys) in 2019 were awarded a Distinction, Merit, Pass or Fail.

## C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

## D. Notice of examination conventions for candidates

Notices to candidates were sent on: 9 October 2018 (first notice), 13 November 2018 (second notice), 14 February 2019 (third notice) and the 13 May 2019 (final notice).

The examination conventions for 2018-2019 are on-line at http://mmathphys.physics.ox.ac.uk/students.

## Part II

## A. General Comments on the Examination

Table 2 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 2: Rank and percentage of candidates with this or greater overall USMs

| Av USM | Rank | Candidates with <br> this USM and above | $\%$ |
| :---: | :---: | :---: | ---: |
| 93 | 1 | 1 | 2 |
| 91 | 2 | 2 | 4 |
| 90 | 3 | 4 | 6 |
| 89 | 4 | 5 | 8 |
| 88 | 5 | 6 | 9 |
| 86 | 6 | 7 | 11 |
| 85 | 7 | 9 | 13 |
| 83 | 9 | 12 | 17 |
| 82 | 12 | 14 | 23 |
| 80 | 14 | 16 | 26 |
| 79 | 16 | 19 | 30 |
| 78 | 19 | 21 | 36 |
| 76 | 21 | 22 | 40 |
| 75 | 22 | 26 | 42 |
| 74 | 26 | 28 | 49 |
| 73 | 28 | 31 | 53 |
| 72 | 31 | 35 | 58 |
| 71 | 35 | 32 | 66 |
| 70 | 42 | 42 | 79 |
| 69 | 43 | 43 | 81 |
| 68 | 44 | 44 | 83 |
| 67 | 46 | 46 | 87 |
| 66 | 47 | 47 | 89 |
| 61 | 48 | 48 | 91 |
| 60 | 50 | 50 | 94 |
| 59 | 51 | 51 | 96 |
| 58 | 52 | 52 | 98 |
| 31 | 53 | 53 | 100 |

## B. Equality and Diversity issues and breakdown of the results by gender

Removed from public version
Oral Presentation All candidates passed the requirement to give an oral presentation on a specialist topic.

## C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 3 and in the Average USM per Formal Assessment graph below. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 3: Numbers taking each paper

| Paper | Number of <br> Candidates | Avg <br> USM | StDev <br> USM |
| :--- | :---: | ---: | ---: |
| Advanced Fluid Dynamics | - | - | - |
| Advanced Philosophy of Physics | - | - | - |
| Advanced QFT | 31 | 68 | 22 |
| Advanced Quantum Theory | 17 | 67 | 10 |
| Algebraic Geometry | - | - | - |
| Applied Complex Variables | 11 | 68 | 21 |
| Collisional Plasma Physics | - | - | - |
| Collisionless Plasma Physics | - | - | - |
| Differentiable Manifolds | 6 | 62 | 16 |
| Disc Accretion in Astrophysics | - | - | - |
| Dissertation (single unit) | 15 | 77 | 6 |
| Dissertation (double unit) | 8 | 83 | 10 |
| Galactic and Planetary Dynamics | 7 | 72 | 12 |
| General Relativity I | 24 | 67 | 12 |
| General Relativity II | 24 | 68 | 11 |
| Groups and Representations | 40 | 78 | 12 |
| Introduction to Quantum Information | 26 | 81 | 12 |
| Kinetic Theory | 12 | 63 | 5 |
| Networks | 17 | 72 | 7 |
| Numerical Linear Algebra | 8 | 69 | 9 |
| Perturbation Methods | 10 | 71 | 8 |
| Quantum Field Theory | 49 | 71 | 14 |
| Quantum Matter | 13 | 73 | 7 |
| Radiative Processes and High Eng. Astro. | - | - | - |
| String Theory I | 19 | 72 | 19 |
| Supersymmetry and Supergravity | 12 | 72 | 19 |
| Theories of Deep Learning | - | - | - |

Average USM Per Formal Assessment


The number of candidates taking each homework completion course is shown in Table 4. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 4: Numbers taking each homework completion course

| Paper | Number of <br> Candidates | Percentage <br> completing course |
| :--- | :---: | ---: |
| Advanced Fluid Dynamics | - | - |
| Advanced Quantum Field Theory | - | - |
| Astroparticle Physics | - | 100 |
| Collisionless Plasma Physics | - | - |
| Conformal Field Theory | 12 | 100 |
| Cosmology | 13 | 100 |
| Disc Accretion in Astrophysics | - | - |
| Galactic and Planetary Dynamics | - | - |
| Group and Representations | 40 | 100 |
| Kinetic Theory | - | - |
| Nonequilibrium Statistical Physics | 14 | 93 |
| Quantum Field Theory in Curved Space Time | 12 | 83 |
| Quantum Matter | - | - |
| Renormalisation Group | 15 | 100 |
| Soft Matter Physics | 9 | 100 |
| String Theory II | - | - |
| Supersymmetry and Supergravity | - | 100 |
| Symbolic, Numerical and Graphical Scientific Programming | 8 | 100 |
| The Standard Model and Beyond I | 9 | - |
| The Standard Model and Beyond II | - | - |
| Topics in Soft and Active Matter Physics | - | - |
| Topics in Quantum Condensed Matter Physics | - | 100 |
| Topological Quantum Theory | 30 |  |

## D. Assessors' comments on sections and on individual questions

## Advanced Fluid Dynamics

Question 1: Part (a), which was bookwork, was done well by most, although there was a lack of clarity in some minds regarding the precise mathematical assumptions (ordering of fields with $\beta$ and Mach number) behind the iMHD equations.

Part (b) was done well by most.
Part (c) did not present much difficulty either, although there was some confusion as to whether $\partial_{i} M_{i j}$ represented magnetic pressure or tension.
Part (d) was elementary, but some nevertheless managed to fail to take the trace of $\delta_{i j}$ correctly.

Part (e), being straight linear theory, should have been easy, but defeated all. Students could not get to the dispersion relation and none realised that this task might be simplified by introducing the displacement vector (despite this having been covered in the lectures and homework).
Part (d) None noted that while magnetic fields could reverse direction on small scales (leading to dissipation), polymers strands did not have that problem.

Question 2:
Almost all candidates calculated the force by integrating $\sigma \cdot \mathbf{n}$ over the surface of the sphere, as in lectures. The "hence" in the question was thus relaxed to "hence, or otherwise" when marking. Candidates were expected to use $\nabla \cdot \sigma=0$ and the divergence theorem to show that the integral of $\sigma \cdot \mathbf{n}$ over the surface of the sphere equals the integral of $\sigma \cdot \mathbf{n}$ over any larger enclosing surface. By considering a sphere of large radius one can use the large- $r$ approximation of the velocity field to simplify the calculation. by dropping the $\left(r^{-3}\right)$ terms. One ends up needing to integrate $x_{i} x_{j}$ over a unit sphere, which is most easily done by observing that the integral must be a scalar multiple of $\delta_{i j}$, by isotropy, then taking the trace to find the scalar.

Almost all candidates then tried to address (b) using the reciprocal theorem, which was correctly stated, instead of recognising the large- $r$ expansion of the velocity field they were expected to have used in (a). They should also have eliminated $\mathbf{U}$ in favour of $\mathbf{F}$ using $\mathbf{F}=-6 \pi \mu a \mathbf{U}$.

Part (c) was done best. However, many candidates simply asserted that $\mathbf{u}_{1}^{\prime}+\mathbf{u}_{2}^{\prime}=0$, or invoked an unspecified symmetry principle, to show that the centre of the bead-spring pair moves with the velocity of the surrounding fluid. They should have calculated $\mathbf{u}_{1}^{\prime}$ and $\mathbf{u}_{2}^{\prime}$ by considering the spring force on each sphere. Various factors of 2 were lost by trying to put the spring force, sometimes called an external force, straight into the $\mathbf{R}$ equation.

Not all candidates identified $\zeta$ as the Stokes drag coefficient $6 \pi \mu a$. For the last part, candidates were just expected to note that $\zeta /(8 \pi \mu R)=3 a / 4 R$.

## Advanced Quantum Field Theory

All questions were to be attempted. The standard of answers was good in general, and students did not struggle with one question in particular; the average marks for each of the questions were rather similar. Further comments follow below, focussing on elements that some students had issues with.

Question 1) a) i) was basic bookwork, and the majority of students got this right. There were many elements to a) ii) to get right (colour factors, minus sign for ghost loop, knowing when/how to apply PV reduction, getting the algebra right for the loop calculation), and most students did not got every part of this correct. The majority of the students followed the hint and got the marks for a) iii), and most students identified the correct method for a) iv), even if the stated answer contained an error carried forward from (ii). Most students had no issues with b), which was entirely bookwork.

Question 2) Most students had no issue with a) i) and ii). Many students struggled to prove b) i) in real time, or got lost in the algebra. Marks were lost in b) ii) and iii) by some students due to apparent lack of familiarity with Feynman rules for QED, and the spinor sum procedure. Those who were familiar generally had no major issues. The relative minus sign was often missed between the two diagrams in b) iii), and in some cases the students failed to identify the correct form of the trace for the cross term, and its relation to b) i).

Question 3) a) Was bookwork and was generally answered to a high standard. Some students introduced sign errors when moving to Fourier space or did not quite get the final inversion right to produce the propagator. For b) i) quite a few students failed to identify the correct $S O\left(N^{2}\right)$ symmetry for the Hermitian case. ii) and iii) were answered without any major common issues. Where marks were dropped for iv), this was due either to
students not identifying the correct method for substituting the expression for $\Phi$ into the potential and/or not manipulating the corresponding traces correctly.

## Advanced Quantum Theory

Q1: leniency was given in part (c) in terms of assumptions made on the various parameters, and in 1(d) for alternative ways to argue for the order of the different transitions.

Q2: a full answer for part (d) would note that for ferromagnets the Bogoliubov vacuum $|0\rangle$ is a ground state, but so is $\left[b^{\dagger}(\vec{k}=0)\right]^{N}|0\rangle$ for $N \leq 2 S$ (not on the original solutions but on the amended ones). While full marks were given for the former answer, some candidates gave rather obscure explanations involving some version of the latter; only partial marks were given for such answers unless the explanation was very clear.
Q3: Leniency was shown in part (b) as long as some effort was made to justify the arguments via integration by parts - but full credit was only given if there was a clear explanation for why the boundary condition allowed the neglect of boundary terms when integrating by parts (the 'and hence' in the question was a clear hint that both the definition of the Green's function and the boundary conditions it satisfies were necessary to successfully rewrite the generating functional as required.) In Part (f) there is an additional disconnected double tadpole that should be accepted as an allowed solution; any 2 of the 3 new diagrams were accepted for full marks.

## Collisionless Plasma Physics

There were no issues with the exam. The students did not need any clarifications.

Question 1. The students were able to solve parts (a) and (b). In part (c)(i), all the students missed the existence of particles that cannot reach the origin. Some students thought, incorrectly, that the particle trajectories were "unstable". None of the students were able to find the solution of part (d) even though a very similar question is asked in the first homework.

Question 2. Students were able to solve parts (a)-(d) and part (f), and struggled with the algebra of part (e) (several students failed to neglect $k_{\|}$in one of the terms). Students found parts (g) and (i) harder than expected. In part (h), students were not careful and they did not consider
the possibility of $k_{\|}$being small.
Question 3. Students solved part (a) and part (b), although they spent more time on part (b) than expected. Part (c) and part (d) were partially solved by all the students, but in both parts, the students did not seem to understand important aspects such as the reason why $\omega / \omega_{d i}$ must be positive for instability.

## Collisional Plasma Physics

The exam had two typos:

- In part (a) of problem $3, \partial^{2} u_{i z} / \partial z^{2}$ should have been $\partial^{2} u_{i z} / \partial x^{2}$. The students realized that it was a typo, and it was corrected during the exam.
- In part (d) of problem 2, the collision operator $C\left[g_{e}\right]$ should have been $C_{e e}^{(\ell)}\left[g_{e}\right]+\mathcal{L}_{e i}\left[g_{e}\right]$. Some of the student's mistakes are not attributable to the typo, but others are. I awarded points to the student to correct for the typo.
The rest of the question went as follows:
- Problem 1. All the students did well in this problem in general. There were some errors in the solutions to the different differential equations, and the students did not attempt to discuss the physical meaning of the results at the end of parts (d) and (e).
- Problem 2. Students who attempted this problem, did well in it except for part (d).
- Problem 3. Parts (a) and (b) of this problem were solved by all the students. Part (c) was only partially answered by some students because they failed to see that the gyroviscosity contributed a force in the $x$-direction.


## Groups and Representations

Q1: On the whole, very well done. Some difficulties were encountered in part D, mainly related to how the argument is made in a formally satisfactory manner.
Q2: Attempted by 38 students, average mark 21.5. ted to algebraic errors in the calculation.
Q3: Parts A, C and E were completed by most students who attempted the question. A common problem in part $B$ was that parts of the argument
were left out (such as showing the map is subjective) Most students did not understand how to tackle part D.
Q4: . Parts A, B and C were mostly done fairly completely, with difficulties usually related to simple algebraic mistakes. Only about half of the students managed to write down the representation in Part D.

## Kinetic Theory

Question 1: Part (a) the only common error was asserting that the temperature was the moment with respect to $|\mathbf{v}|^{2}$, rather than $|\mathbf{v}-\mathbf{u}|^{2}$. A few candidates mistakenly took "moments" with respect to $\mathbf{u}$ and $\theta$, which are functions of ( $\mathbf{x}, t$ ) only, to try to show the conservation properties.

Part (b) several candidates wrote explanations starting from (*) of why one would be interested in the displayed integral, then just asserted the required result. One gave only a perturbative result based on expanding for $f$ close to $f^{(0)}$.
Part (c) A few candidates left $g^{(0)}$ and $h^{(0)}$ as unevaluated integrals. The most common error was not realising that $\theta$ depends on both $g$ and $h$, or not giving expressions for $\rho, u, \theta$ in terms of $g$ and $h$ at all.

Part (d) This caused more difficulty than expected. A few candidates wrote down general results for the original 3D equation (*) for $f$ rather than starting from the system for $g$ and $h$ as instructed. Some who had answered (a) correctly then forgot that $\theta$ was a moment with respect to $|\mathbf{v}-\mathbf{u}|^{2}$, not with respect to $|\mathbf{v}|^{2}$. While one can treat the contributions to the energy density from $g$ and $h$ separately at first, it is not true that they are separately conserved under collisions, as $\theta$ depends on both $g$ and $h$.

Question 2: This entire question proved to be extremely challenging to students.

Part (a) was pure bookwork on which everyone ought to have been able to get full marks. Quite a few did, but those who did not, did not because they were unable to explain the logic behind the dispersion relation $\epsilon(p, k)=0$, even if they might have vaguely understood it. There was also some unnecessary explanation of irrelevant material memorised from the lectures.

Part (b). While several students realised that the double Lorentzian had its own poles, a surprising number did not. Of those who did realise, only some performed the integral using the Cauchy residue theorem.

Part (c). Students were required to solve the biquadratic dispersion relation
(formula (3), already given to them in the question so their ability to use it would not be predicated on their ability to derive it in part (b))-and then take the limit of $u_{b} / v_{p} \ll 1$ carefully.
Part (d). This required looking at the roots of (3) and their dependence on k.

Part (c). Nobody got this far.
Question 3: Candidates scored highly on parts (a)-(c) but nobody got more than half marks on part (d). Few candidates thought to write down Hamilton's eq of motion for J, and nobody thought to solve this by Laplace transforms. Several candidates got a mark for part (e), but nobody made the hoped-for comment about the connection with the conventional derivation of a Fokker-Planck equation by Taylor expanding the integrand of the master equation.

The physics and maths of part (d) are simplicity itself by comparison with some of what's involved in parts (a) to (c). The problem was that parts (a) - (c) involve reproducing the lecture notes, while part (d) requires independent thinking. The candidates have worked diligently, but don't yet use their tools confidently.
The answers to part (a) suggested that the fundamental conceptual split into a mean-field model and fluctuations about it wasn't as clear in candidates' minds as it should be. Several candidates seemed to think that the evolution of the mean-field model hinged on evaporation, and implied that the mean-field model had a Maxwellian velocity distribution.

## Quantum Field Theory

Question 1: 1a: Many students did not find a simple way of deriving the Hamiltonian, and ended up with mistakes that either made the next part too simple or too complicated.
1b: For students that had incorrect expressions in 1a, they were almost always deriving the wrong Equation of Motion. 1c: Several students did not write down the commuatation relations and therefore did not get full marks for this part.
Question 2: For question 2a, a lot of candidates had difficulty with the vertex with derivatives and count them as a propagator as well.
2 b And to calculate the scattering amplitude, a lot of candidates had difficulties with the vertices with derivative as well. A standard answer expects candidates to formulate Feynman rules with clean notation and thereafter
calculate the amplitude of scattering accordingly.
Question 3: The first part of the this question did not cause any major difficulty. All students seemed to have well in mind the properties of gamma matrices, and then explicit chiral representation which took most of them through parts A-D without any trouble. Part E cause more difficulties however. Only roughly $1 / 2$ of the students properly managed to write the effect of a Lorentz transformation on the Dirac action. Out of these, only a part could carry the calculation to the end to compute the Lorentz transformation of chiral components.

## Quantum Matter

This exam was a bit harder than it was in past years. Time seemed to be a factor for some fraction of the students.

There was an error on the exam. In question 2 c and 2 d the information should have been given that one should focus only on the case of $N=2$ fermions. This error was spotted before the exam was given and an erratum was distributed with the exam, so this should not have caused any trouble. (For the record, this information was in an early version of the exam but somehow got lost in the editing!)
Q1:
1a. All students could give a good definition of bose condensation, and most could say a few words about how it differed from superfluidity. Marks here were high.
1b. Most students could figure out that in this case one would only have bose condensation in $d>4$. Most marks were high here.

1c. Almost all students knew how to write a Hamiltonian in second quantized form. However, a few lost factors of 2, or introduced spurious factors of $N, V, \hbar, 2$. Marks again were high here.
1d. Most students had no trouble getting to the end result. However, justifications of how one goes from field operators to complex scalar fields were often not correct, or even entirely absent. Mixed results here.

1e. Here is where things got troublesome. Although this was almost identical to a homework problem, very few students managed to derive the excitation spectrum. Many low marks here.
1f. If one didn't properly get the excitation spectrum, it was hard to do the last point, although partial marks were awarded if anyone even wrote
down the landau criterion. Many low marks here (and unfortunately some students who got stuck on 1e didn't even try this part).

Q2.
2a. Almost all students had no trouble defining Hatree Fock
2b. ... and very little trouble explaining how Hartree-Fock works. Marks on these two parts were high.
2c A two site model with two electrons seemed to stump quite a few students. Even when we turned off the interaction - this reduces to a noninteracting electron system, and still students got stuck. The interacting (but no hopping) case was also puzzling to many - but again this should have been fairly easy since the Hamiltonian is so simple. Marks were mixed here. There were a few perfect scores on this part, but surprisingly few.

2d. A few students did realize why nondegenerate ground state along with spin symmetric Hamiltonian implies a spin singlet. A few got close to the right argument for why the form of the $f$ operators is as given (although fewer actually got the argument perfect). A few did identify the form of the ground state wavefuction, but only two correctly showed that HF is never exact when $t$ and $U$ are both nonzero. I admit this last part of the questions required some careful thought, and I expected few to get full marks.

## Radiative Processes and High Energy Astrophysics

The paper was taken by a small number of candidates and the answers were of very high quality.

Particularly impressive was that all candidates answered the three-level atom question perfectly. This topic has had an anecdotal reputation for difficulty.
The answers to the question on particle acceleration showed a few occasions where the candidates may not have has a full grasp of the discussion in the lectures. In particular, the point was missed that the synchrotron emission from protons is much weaker than from electrons in the same magnetic field. Thus a jet with such "hidden" protons would require either super-Eddington accretion or would leave an abnormally massive remnant black hole

## Supersymmetry and Supergravity

The students did very well overall. The first two-third of questions (1) and (2) were straightforward in principle, and everybody did well, except on the question on the Lagrangian of SQCD.

Question 1(f) required checking SUSY algebra on superspace. One needed to do the computation carefully not to "forget" any terms. The challenging part was the commutator $[M, Q]$.
On question 1(h): the aim was to show that the $4 \mathrm{~d} \mathcal{N}=4$ and $4 \mathrm{~d} \mathcal{N}=3$ supermultiplets have the same field content. While the $\mathcal{N}=4$ multiplet is a proper SUSY multiplet, with $\mathcal{N}=3$ we need to consider two proper supermultiplets, CPT conjugate of each other, and this physical $\mathcal{N}=3$ multiplet has then the same field content as the $\mathcal{N}=4$ one, as in the branching rules given in the question. Most students had difficulty assigning the $U(1) \subset U(3)_{R} R$-charge. One needed to realize that $Q$ has $R$-charge -1 , and that the $U(1)$ R-charge of an helicity $\lambda=1$ particle should be zero, because it's a real vector field and thus it cannot carry $U(1)$ charge.

On question 2(f): Some students had some confusion on the meaning of $W$. Recall that $V=\left|\partial_{\phi} W\right|^{2}$ and only the derivative of $W$ is then "physical," when it comes to looking for SUSY vacua. In particular, we need $\partial_{\phi} W=0$ in a SUSY vacuum, not $W=0$.

## C3.3: Differentiable Manifolds

Question 1: Attempted by most candidates. Part (c) had quite a lot of things to cover and even candidates who knew what they were doing tended to lose a few marks by missing things out (e.g. by not explaining why $X$ is Hausdorff, and second countable, which is why f-1(y) was supposed finite or countable).

Part (d): The answer is that f is not a covering map (because of behaviour over 1 in $Y$ as 0 is not in $X$ ), but it is a local diffeomorphism. You could have inferred the first from the question just on logical grounds: as (c) shows that covering maps are local diffeomorphisms, if $f$ were a covering map, then there would be no point in the examiners also asking if $f$ is a local diffeomorphism. But almost everyone said f is a covering map, and a surprising number did not answer the question about local diffeomorphisms.
Question 2: (a),(b) were bookwork and done well. Candidates found the first parts of (c),(d) difficult ((c)(i) needed an algebraic trick which most did not spot, though many got part marks; in (d) few could explain that
$S^{1}$-invariant $k$-forms $\alpha$ on $S^{1} \times Y$ are $d \times \wedge \beta+\gamma$ for $\beta, \gamma k-1, k$-forms on $Y$ ), but for the second parts marks were obtained.

Question 3: The least popular question. For (a), some candidates did not know the definition of orientations on manifolds in terms of orientations on tangent spaces. Part (d) was difficult, and candidates who gave up and did not attempt it received at most 13 marks.

## C3.4: Algebraic Geometry

Almost all candidates chose exercise 1, after which as second option exercise 3 was about twice as popular as exercise 2.
Exercise 1: (b) almost all candidates did not take the closures of the Csets, confusing the condition of being relatively closed in the qpv X with being closed in the ambient projective space; (c) surjectivity seems to have stumped many candidates even though it was clear due to there being a quotient map on coordinate rings for subvarieties; (d) frequent mistake: candidates used isomorphisms $\mathrm{f}, \mathrm{g}$ to identify the qpvs $\mathrm{V}, \mathrm{W}$ with affine varieties $A, B$, and then took $A$ intersect $B$, but $A$ intersect $B$ is in general unrelated to V intersect W. Only very few candidates used the map (fxg) applied to ((VxW) intersect (Diagonal)), and the fact that (fxg)(Diagonal) is closed in AxB. Exercise 2: (a) candidates often wrote the definition of tangent space for an affine variety in terms of a vanishing set, rather than the intrinsic definition needed for a projective variety or a qpv; (d) candidates sometimes did not see that one had to consider the Pluecker embedding, in order to justify why the map was a morphism. Exercise 3: (a) many candidates did not explain to which algebra g ,h belong, when writing $f=g / h$, in the definition of regular function; (b) a lot of confusion by candidates caused by using the coordinates $x_{j}$ (with $x_{i}$ omitted), rather than $x_{j} / x_{i}$ on the affine charts $U_{i}=\left(x_{i}\right.$ not zero). Candidates erroneously thought that the function was therefore a polynomial in the $x_{j}$ on $U_{i}$ independent of $x_{i}$, and therefore the polynomial was independent of all coordinates $x_{i}$, hence constant! (c) Some candidates stated what algebras are involved, but without saying how the equivalence maps objects and morphisms; (d) most candidates forgot that the Veronese embedding allows one to prove that $P^{n}(F)$ is affine (proved in the notes, and arises in a homework exercise)

## C5.6: Applied Complex Variables

There were no errors on the paper. There was a slight possibility of confusion on Q1. The question suggested integrating $d z=d \zeta$ to show a result,
when $\zeta$ had not been defined. I had realised this in advance of the paper being sat, but decided against an announcement because (i) $\zeta$ has a standard definition used throughout lectures which most candidates would just assume; this indeed turned out to be the case; (ii) any other definition of $\zeta$ will also work;
(iii) an announcement would be difficult to phrase and likely to confuse candidates.
In the event only one candidate raised a query as to the definition of $\zeta$, and I responded by saying he should decide what to take zeta to be. I would expect the raw marks to be a good approximation to USM. There was a good range of $\alpha$ and $\beta$ marks for each question.

## C6.1: Numerical Linear Algebra

This paper was largely well-attempted. It was surprising that several candidates attempted all 3 questions despite the clear guidance in the rubric.

Most candidates attempted question 1 on matrix factorizations, with a range of scores. Few saw the point of the final part (f) where in particular calculation of the smallest singular value was only correctly done by very few. Too many candidates were happy to query that $L_{1} U_{1}=L_{2} U_{2}$ implies $L_{1}=L_{2}, U_{1}=U_{2}$ in part (c) without adequate proof.

Question 2 on stationary (simple) iteration was attempted by just over half of the candidates with a range of scores including one full marks. In the final part (c) too many candidates were too quick to introduce $B^{-1}$ thereby making in almost impossible to apply simple diagonal dominance arguments.
Question 3 on Krylov subspace methods was attempted by under half of the candidates, but attracted (in general) higher marks. Again the final part (c), though well done by some, caused difficulty.

## C7.4: Introduction to Quantum Information

## Question 1

Well done question. Some students struggled with part (b) and the calculations in part (d). Many students failed to provide physical interpretation of the results in part (g).

## Question 2

This was the most popular question on the paper. Parts (a) and (b) were
standard problems and did not pose much difficulty. Some students failed to spot the linearity of equations in part (c). In part (d) most marks were lost for not estimating the imaginary part of the trace. Good attempts at part (e).

## Question 3

The bookwork in parts (a) and (b) caused no problems. Most marks were lost in parts (c) and (e).

## C7.5: General Relativity I

Q1: This question was the least popular. Parts a to c were mostly bookwork and well done. Candidates had difficulty proving the Bianchi identity for the field strength in part d. Most candidates were unable to complete part f , and there were no correct answers for part g .

Q2: Parts $a$ and $b$ were mostly well done, except for a few algebraic errors. Some candidates used the Euler-Lagrange equations for part a but did not state why this was equivalent to varying the action. In part d, a handful of candidates were able to use the conserved quantities to identify the new coordinates, though few correctly identified the region of $(T, X)$ plane covered by the old coordinates.

Q3: This question attracted the most attempts. Candidates lost marks in part a by not specifying spherical symmetry and in part b by not showing that the Lagrangian is constant on the geodesic. Part $d$ was attempted by many but completed correctly by few. Parts e, f and g were well done by those who attempted them.

## C7.6: General Relativity II

Question 1: The question was attempted by less than half of the candidates. Part a) was done very well in general. Most of the candidates wrote down the correct definitions. In part b), many candidates explained well why the metric given in the problem is the most general form of the spherically symmetric metric. Several candidates attempted the coordinate transformations but most of them could not reach the conclusion except for few candidates. In part c ), most of the candidates only show $B=B(r)$, and only one candidate proved that $A$ can also be chosen to be $A=A(r)$. None of the candidates proved that the equation also satisfies $B_{r r}=0$.

Question 2: It is pleasing to see most of the candidates can compute Christoffel symbol of a spherically symmetric metric. It is also observed that most of the students had difficulty with straightforward but long calculations, for example, computing the components of Ricci tensor of Vaidya metric. Most of the students had managed to compute just only one non-vanishing component of Ricci tenor, although the question was meant to calculate all components. Also, all examinees taking question $2 b)$ had not shown the vector field tangent to the out-going energy flux in Vaidya space-time obeys null geodesic equation. Some students had not understood the definition of a space-like hyper-surface properly, as attempts were made to show that the normal vector of such a hyper-surface is space-like. Question 3: Nearly all students attempted this question. Most students did well on the bookwork parts a) and b). Many candidates came up with a correct strategy to solve part c), but nearly everyone struggled with the longer computation that requires correct manipulation of the metric components of Kerr. The first part of d), finding $\lambda_{+}$such that $\eta_{+}$is null on the horizon $r=r_{+}$, was worked out correctly by a good proportion of the candidates, however nearly no one showed that $\eta_{+}$is time-like for all $r>r_{+}$. Finally, part e) seemed easier again for most of those candidates who attempted it and most of those scored good points here.

## E. Comments on performance of identifiable individuals

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## F. Names of members of the Board of Examiners

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Prof Artur Ekert (Mathematical Institute, University Of Oxford)
Prof Lionel Mason (Mathematical Institute, University Of Oxford)
Dr Dmitri Pushkin (Department of Mathematics, University of York)
Prof Alex Schekochihin (Department of Physics, University of Oxford;
Chair)
Prof Daniel Waldram (Department of Physics, Imperial College London)

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Prof Fernando Alday
Dr David Alonso
Dr Anthony Ashmore
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Dr Richard Berry
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