Examiners' Report: Final Honour School of Mathematical and Theoretical Physics Part C and MSc in Mathematical and Theoretical Physics Trinity Term 2017

October 27, 2017

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers		Percentages %	
	2017	(2016)	2017	(2016)
Distinction	31	(18)	76	(85.71)
Pass	10	(3)	24	(14.29)
Fail	0	(0)	0	(0)
Total	41	(21)	100	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**
  No vivas were held.

- **Marking of scripts.**
  All dissertations and mini-projects were double-marked, after which the two markers consulted in order to agree a mark between them.

All written examinations and take-home exams were single-marked according to carefully checked model solutions and a pre-defined marking scheme which was closely adhered to. A comprehensive independent checking procedure is also followed.

B. New examining methods and procedures

The process for calculating the final classification was simplified for 2016–17 to make it more transparent. The revised process for calculating the $\overline{USM}$ was agreed by the Joint Supervisory Committee and published in the examination conventions at the start of the year.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

Notices to candidates were sent on: 13 October 2016 (first notice), 21st November 2016 (second notice), 15th February 2017 (third notice) and the 5th May 2017 (final notice).

The examination conventions for 2017 are on-line at http://mmathphys.physics.ox.ac.uk/students.

Part II

A. General Comments on the Examination

Table 2 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 2: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
96	1	1	2.44
95	2	2	4.88
91	3	4	9.76
90	5	5	12.20
88	6	7	17.07
87	8	8	19.51
86	9	9	21.95
85	10	10	24.39
83	12	12	29.27
82	13	13	31.71
80	14	15	36.59
79	16	16	39.02
78	17	20	48.78
77	21	23	56.10
75	24	25	60.98
74	26	27	65.85
72	28	28	68.29
71	29	31	75.61
69	32	33	80.49
66	34	34	82.93
65	35	36	87.80
64	37	37	90.24
63	38	39	95.12
60	40	40	97.56
52	41	41	100

B. Equality and Diversity issues and breakdown of the results by gender

This section has been removed the from public report, as the cohort contained fewer than 6 candidates

Oral Presentation All candidates passed the requirement to give an oral presentation on a specialist topic.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 3. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 3: Numbers taking each paper

Paper	Number of Candidates	Avg USM	StDev USM
Advanced Fluid Dynamics	7	65.43	19.69
Advanced QFT	10	73.40	18.49
Advanced Quantum Theory	13	86.00	14.98
Algebraic Geometry	1	-	-
Algebraic Topology	1	-	-
Applied Complex Variables	2	-	-
Astrophysical Gas Dynamics	4	-	-
C3	1	-	-
Collisional Plasma Physics	3	-	-
Collisionless Plasma Physics	9	87.78	9.47
Differentiable Manifolds	4	-	-
Galactic and Planetary Dynamics	3	-	-
General Relativity I	19	69.63	10.36
General Relativity II	17	71.88	
Geometric Group Theory	1	-	-
Geophysical Fluid Dynamics	3	-	-
Groups and Representations	26	75.92	13.56
Introduction to Quantum Information	22	75.86	10.64
Kinetic Theory	12	72.25	12.74
Networks	9	69.00	5.76
Nonequilibrium Statistical Physics	8	89.25	7.07
Numerical Linear Algebra	1	-	-
Perturbation Methods	7	67.71	11.13
Quantum Field Theory	39	70.25	13.67
Quantum Matter	11	70.90	13.46
Radiative Processes and High Energy Astrophysics	2	-	-
Statistical Mechanics	4	-	-
String Theory I	16	66.56	12.89
Supersymmetry and Supergravity	16	80	10
Dissertation (single unit)	9	74.44	7.1
Dissertation (double unit)	10	81.20	7.33

The number of candidates taking each homework completion course is shown in Table 4. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 4: Numbers taking each homework completion course

Paper	Number of Candidates	Percentage completing course
Astroparticle Physics	19	100
Astrophysical Gas Dynamics	4	-
Beyond the Standard Model	6	100
Conformal Field Theory	16	100
Cosmology	6	100
Group and Representations	22	100
Introduction to Gauge-String Duality	3	-
Nonequilibrium Statistical Physics	8	100
Non-perturbative Methods in Quantum Field Theory	5	-
Quantum Field Theory in Curved Space Time	11	90.9
Quantum Matter	11	100
Radiative Processes and High Energy Astrophysics	2	-
Soft Matter Physics	7	100
String Theory II	9	100
The Standard Model	1	-
Topics in Quantum Condensed Matter Physics	4	-
Topics in Soft and Active Matter Physics	3	-
Topological Quantum Theory	21	100

D. Assessors' comments on sections and on individual questions

Advanced Fluid Dynamics

Question 1
This question proved to be more challenging than was expected in nearly all of its parts.

(a) A surprising number of students failed to recognise that proving $\nabla\cdot\mathbf{B} = 0$ was an important step in ascertaining the legitimacy of the proposed field and another set did not appreciate that $\mathbf{B}\cdot\nabla\alpha = \mathbf{B}\cdot\nabla\beta = 0$ proved that α and β stayed constant on field lines and that fixing the configuration of the footpoints was equivalent to setting α and β values on the boundary.

(b) It proved challenging for nearly everyone to figure out that the constraint of fixed α and β at the boundary simply meant that in applying the variational principle one had to vary α and β subject to those variations being zero at the boundary. However, one candidate did get full marks on this question and another had the right idea, nearly got there, but gave up —thus giving the proof of principle that it was not impossible to solve this part of the question.

(c) This part went smoothly for nearly everyone, except for the minor sin of not stating explicitly that linear force-free field was one with constant λ. But nearly all used that fact implicitly and so lost no marks.

(d) What was an extremely straightforward problem in solving an ODE went wrong for far too many candidates. Several candidates got themselves tangled up in algebra by not realising that the required field could be obtained by assuming that it was $\exp(-\gamma z)$ times a function of x only. Applying boundary conditions to a sinusoidal solution of a wave equation and hence working out constants also proved more challenging than it ought to have been for 4th-year students.

Question 2
Part (a) was done well by almost all candidates. One candidate used an incorrect argument to replace $\mathbf{R}\cdot\mathbf{n}$ by $\mathbf{R}$, instead of relating an expression for the force to an integral of $\boldsymbol{\sigma}\cdot\mathbf{n}$.

There was an error in part (b). The hint for the Brownian force should have been $-k_B T\,\nabla_{\mathbf{r}_i}(\log\psi)$, as in lectures, not $-k_B T\,\nabla_{\mathbf{r}_i}\psi$. This error was spotted by a candidate, and a correction announced, within the first 30 minutes. No candidate lost marks due to this error. All candidates found the equation for $\mathbf{R}$, up to some lost factors of 2. Several candidates were careless about $(\mathsf{I} + \alpha\boldsymbol{\sigma})$ and its inverse being matrices, which must remain on the left of the vectors they multiply.

The integration to find an evolution equation for $\mathsf{C} = \langle\mathbf{RR}\rangle$, a useful intermediate step, caused more problems. One candidate did not integrate at all, and another lost many terms by trying to integrate in vector notation rather than integrating by parts using suffix notation.

Only two candidates used $\overset{\nabla}{\mathsf{I}} = -(\nabla\mathbf{u}) - (\nabla\mathbf{u})^{\mathsf{T}}$ to get the strain rate term in the equation for $\boldsymbol{\sigma}$. One candidate tried instead to get $(\nabla\mathbf{u}) + (\nabla\mathbf{u})^{\mathsf{T}}$ from the Brownian term in the evolution equation for $\langle\mathbf{RR}\rangle$. It is useful to note that $(\mathsf{I} + \alpha\boldsymbol{\sigma})$, $\boldsymbol{\sigma}$, and C all commute, because they are all linear combinations of I and C. A few candidates correctly obtained $\lambda = \zeta/(4H)$ and $\mu = nk_BT\zeta/(4H)$, but there were many calculational errors.

No candidate observed that the definition of $\boldsymbol{\sigma}$ implies $\boldsymbol{\sigma} = 0$ when $\nabla\mathbf{u} = 0$.

In part (c), the derivation of quadratic equations for σ_{11} and σ_{22} was done well by almost all candidates, although one lost factors of λ.

Several candidates asserted that $\sigma_{11} \sim -\mu/\lambda$, which only holds for $\alpha = 0$ and $\lambda\gamma \gg 1$. Only one candidate commented on the stress then being negative.

Noone wrote explicitly that $\sigma_{11} \geq 0$ when $\alpha > 0$, but when $\alpha = 0$ the elongational stress σ_{11} increases to infinity when $\gamma\lambda = 1/2$, then becomes negative (and hence mechanically unstable) when $\gamma\lambda > 1/2$.

Advanced Quantum Field Theory

The candidates were asked to answer all 3 questions. The average mark for these questions were 20.7, 14.9 and 16.0 out of a possible 25 marks. Two candidates did not make significant attempts at the second and third questions and removing them the average marks become 21.6, 17.4 and 19.4, demonstrating that the remaining candidates did all questions reasonably well.

Question 1 involved a mix of bookwork and calculation and was generally well done. Question 2 started with a apart on more formal bookwork and this was quite poorly done, explaining why the average mark was lower. The calculation part was well done. As for question 1, the mix of bookwork and calculation was generally well done in question 3.

Advanced Quantum Theory

I think this exam was fine for MMathPhys.

The following comments are regarding the marking scheme. Part (a). .25 was lost if the student did not mention that the transfer matrix is symmetric therefore diagonalizable. If the students stopped at $Z = Tr[T^N]$ this was a 2 point deduction. However, if the diagonal form of T was written with this, then I gave full credit.

As mentioned above algebraic mistakes were generally .25 deductions. Errors that followed from prior mistakes were not deductions if the process was properly followed. However additional deductions could be made if the student ended up with an answer that they should have spotted as nonsense as a result.

In the first 6 mark section, a lot of students ended up with 1 on the off diagonal. If they seemed to set it up correctly and somehow forgot to add 1+1+1 I deducted .5. If not, I would deduct 1. Usually the deciding factor was whether they had an expression for T written as a sum over tau in some way that should have given them the right answer. A huge number of students could not diagonalize the 2x2 matrix, and did not recognize that this can be done by inspection in this case. Errors here were typically only .25 marks. For students who couldnt get started small amounts of credit were given for writing down things that were basically in the right direction, but I capped this type of credit at 1 mark. If the student stopped at the partition function and did not get a free energy I deducted 1.5. In the final expression a lot of students wrote the energy per unit cell rather than per site (.25 deduction). For those who got very little credit, 1 mark was given for f=-(k T/2) log Lambda+. Another .25 deduction if you dropped the sign.

In section c, in the first 2 mark section, a thermodynamic expression not using the transfer matrix would get only 1 mark. The students had to be clear about where U comes from (I think some students just memorized this from a homework) or a 1 mark deduction. .25 deduction if non-normalized eigenvectors were used, or if U did not have the sqrt(2). A correct detailed outline of this section without a full calculation would get 3/5. No answer that gives a nonzero result would get more than 4 since students should have seen that the answer has to be zero.

In section d the first equation was worth 1 mark. .5 for any thermodynamic expression, and .5 for realizing that one cannot just write $Tr[tau^2 T^L]$ in the sigma basis. If an error in part a made the result of the matrix expression come out incorrect here I deducted .5. Again .25 for using an incorrectly normalized U. Max of 3.5 for anything obviously wrong (including expections of 1 or greater).

Collisional Plasma Physics

Only three students took the exam, and all of them performed very well.

Question 1. There were a few problems with the last question of part (c). The statement asked to check whether the result in part (c) was consistent with the assumptions used to derive it. The students only needed to check that $\mathbf{w} \sim u_{\alpha 0}\sqrt{\nu t}$, but some of the responses were tangential. One of the students did not calculate the constant N, and another did not realize that $\nu_{\|}$ and $\nu_{\perp}$ should be evaluated at $u_{\alpha 0}$ and not at $|\mathbf{w} + u_{\alpha 0}\hat{\mathbf{x}}|$.

Question 2. All the students solved this problem.

Question 3. This was the question that was more difficult for the students. Several students estimated incorrectly the size of the collisional force between electrons and ions because they forgot the thermal force. One of the students failed to give the right estimate for the ion viscosity. Another student was confused about how to treat terms of different order in the same equation. For example, in the total-momentum equation the pressure gradient dominates, leading to the result that it must be small since it cannot be much bigger than every other term. However, this does not mean that the pressure gradient is exactly zero, and it cannot be neglected completely in the total-momentum equation.

Question 4. All the students solved this problem.

Collisionless Plasma Physics

The cohort was strong, and it showed both in the homework and the exam.

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Question 1. In relative terms, this was the question in which the students performed worse. There were two main issues with the students' answers to part (a):

- In some cases, the problems arose when the students tried to prove quasineutrality, or when they justified neglecting the displacement current in Ampere's equation. In both cases, the justification is that in kinetic MHD the plasma is assumed to be non-relativistic. I decided to take points off if the student used that the electric field is zero to justify quasineutrality and the neglecting of the displacement current.

- An important result for part (a) is that the average parallel velocity $u_{s\|}$ only depends on the radial coordinate r. The velocity $u_{s\|}$ is an integral of the distribution function and it is proportional to B. As a result, if the distribution function and the magnetic field magnitude B only depend on r, $u_{s\|}$ only depends on r. If the student used symmetry or other similar argument to justify that $u_{s\|}$ only depends on r instead of the fact that $u_{s\|}$ is completely determined by the distribution function, I took points off. If the magnitude of the magnetic field B was not mentioned, I took points off as well.

One of the students forgot to show that the distribution function provided in the statement satisfied the lowest order drift kinetic equation. Another student decided to show that the distribution function satisfied an even higher order drift kinetic equation. They were lucky that in this configuration, the provided distribution function does indeed satisfy the higher order equation, but in a more general configuration, one would have needed to correct the distribution function with a small extra piece.

Question 2. There was an error in the statement of this question: the lower limit of the integral in equation (3) should have been r_c instead of r_0. The students noticed the error and it was corrected during the exam. All the students performed well in this question, probably because there is a very similar question in the homework. There were minor errors, mostly when taking the limit at infinity of the hyperbolic tangent.

Question 3. There was two small errors in the question: $k_\|$ in equation (8) should be k_z, and after equation (7), L_{T_i} and η_i are defined even though they do not appear anywhere in the problem. These errors were noticed by the students and corrected during the exam. There were a few students that lost points in this problem because they were not careful deriving equation (7). In the definition of the plasma dispersion function, the sign of the imaginary part of ω (and not of ω/k_z) matters. For this reason, it is important to conduct the operations in such a way that the final plasma dispersion relation depends on $\omega/|k_z|$ and not on ω/k_z. Another common mistake for which I took points off was that the students did not consider the signs of k_z and k_y when determining the values of $\mathrm{d}u_{i\|}/\mathrm{d}x$ for which the plasma is unstable.

Question 4. Part (d) of this question caused problems for many students because the statement was confusing. In the statement, the students are asked to assume that $|\omega - \Omega_e| \ll \Omega_e \sim \omega_{pe}$. The frequency ω_{pe} is a function of x that ranges from 0 to a value of the order of Ω_e, so the students were not expected to assume that $|\omega - \Omega_e|$ was always small compared to ω_{pe}, but the statement was clearly confusing. The students did not ask about this part of the question, so the statement was not clarified during the exam. The problem with assuming that $|\omega - \Omega_e| \ll \omega_{pe}$ for every x is that one of the three cut-off densities is found by setting $\omega_{pe} \sim |\omega - \Omega_e|(k_{\|}c/\Omega_e)^2$, and in part (a) students show that one would need to launch a wave with $k_{\|}c/\Omega_e \sim 1$. For this reason, several students were unable to find one of the three cut-off densities. I awarded full points to any student that could not find one of the three cut-off densities because of assuming that $|\omega - \Omega_e| \ll \omega_{pe}$. I only took points off when the students incorrectly considered that $k_\perp^2$ could be complex, or when the students failed to find the two cut-off densities that could be obtained by assuming $|\omega - \Omega_e| \ll \omega_{pe}$.

Galactic and Planetary Dynamics

Three candidates took the Galactic and Planetary Dynamics course. They were assessed by a miniproject in which they were asked to (i) carry out some simple calculations for the simple pendulum problem; (ii) use Lie transform methods to develop the first- and second-order perturbative solution; (iii) to derive the Deprit series for the general case, showing how this can be used to construct "superconvergent" expansions. This was a challenging project: the calculations required in part (ii) were lengthy and the Lie transform method itself is quite subtle. Only one candidate successfully constructed the second-order transformation in part (ii). In contrast, parts (i) and (iii) were more straightforward. All three candidates produced reports of distinction, with the best being almost perfect.

General Relativity II

Geophysical Fluid Dynamics

Mix of solid and strong attempts.

Q1. Marks were lost in a) over derivation of pressure term in the lower layer. and (a) setting the initial potential vorticity, boundary conditions and final interpretation of alternative configurations.

Q2. Strong performance on this question, with marks dropped for missing units, algebraic slips and interpreting signs of wavenumbers in (d)

Q3. a, b done well barring algebraic errors. Candidates appeared to run out of time on later parts

Groups and Representations

Q1) A question on some general group theory and representations of Z4 which was tackled by 22 students, with a strong average of 20.7/25. This question was testing some of the basic knowledge so the high average is expected.

Q2) A question on one of the dihedral groups and its representations tackled by 24 students, with an average result of 18.3/25. This was a somewhat more difficult question which required efficient calculations in parts.

Q3) A question on the group SU(6), its algebra and some of its representations, tackled by 15 students. This was probably the most difficult question with an average of 14.1/25.

Q4) A question on the Lie algebra G2, some of its representation and its A2 sub-algebra, testing the ability to work with roots and weights. This question was attempted by 24 students with an average result of 20.8/25 and could perhaps have been slightly more difficult.

Kinetic Theory

Q1) Section (a) was mostly done well. Several candidates confused energy and temperature, taking moments with respect to $|\mathbf{v}|^2$ instead of $|\mathbf{v} - \mathbf{u}|^2$. Two candidates introduced a multiple-scales expansion of f, which is unnecessary.

Section (b) was done well by almost all candidates. When asked to interpret the peculiar velocity, several candidates wrote something about a "flow velocity" or just "velocity" relative to the mean velocity, with no mention of the particle velocity.

Section (c)

One candidate treated $\mathbf{u}$ as spatially homogeneous, despite the explicit $\mathbf{x}$-dependence in $\mathbf{u} = \mathbf{x} \cdot \mathsf{A}$. A few candidates dropped the contribution from $\mathbf{u} \cdot \nabla\mathbf{u}$ with no justification about it vanishing for a shear flow.

Many candidates tried to derive mass and momentum conservation equation by integrating the displayed equation over $\mathbf{v}$, still treating $\mathbf{x}$, $\mathbf{v}$, t as independent variables, and trying to take x and t derivatives outside the integrals. It is much easier to integrate with respect to $\mathbf{w}$. Only two candidates realised that $\int \mathbf{w}F\,\mathrm{d}\mathbf{w} = 0$ is the defining property of the peculiar velocity, and that this property is preserved by the equation one obtains from the moment with respect to $\mathbf{w}$ of the displayed equation.

The energy equation was found easier, with almost all candidates multiplying by $\frac{1}{2}|\mathbf{w}|^2$ and integrating with respect to $\mathbf{w}$. Several candidates obtained an incorrect expression for the pressure tensor with an extra isotropic term, through not noticing that the isotropic term involving $\mathrm{Tr}\,\mathsf{A}$ vanishes, leaving $P_{ij} = \int w_i w_j F\,\mathrm{d}\mathbf{w}$ as expected.

Several candidates who attempted the last part used the Euler form $\mathsf{P} = \theta\rho\mathsf{I}$ instead of the Navier–Stokes form $\mathsf{P} = \theta\rho\mathsf{I} - \mu(\mathsf{A} + \mathsf{A}^{\mathsf{T}})$ of the pressure tensor. Some assumed a symmetry, or antisymmetric, property of A that does not generally hold. None recognised that $\mathsf{A} : \mathsf{P}$ represents viscous heating, or the work done against shear (as distinct from against an isotropic pressure).

Q2 a) Most students obtained full marks on this part. They know how to take moments of a kinetic equation. The only issues were trivial algebra lapses (signs etc.) b) A surprisingly large number of students failed to realise that the electric-field perturbation must be obtained from the Poisson equation, in terms of δn . A smaller, but still surprising, number thought they were deriving some form of sound waves, rather than Langmuir waves. It appears that at least in some instances, the "pattern recognition" mode of thinking, triggered by the presence of an adiabatic exponent and pressure gradient, overrode physical understanding of plasma oscillations. c) With only a few exceptions, students adequately completed the standard bookwork calculations leading to δf, although some included what was perhaps too much unnecessary (irrelevant) detail. Most were able to identify ballistic/phase - mixing term arising from the initial condition, but fewer realised that the resonant part of also contained a phase mixing term and fewer still that resonant part would develop, on intermediate time scales, a δ- like peak (Van Kampen mode) d) Many appeared to run out of time on this part (probably on account of inefficient handling of part c). Only a few had a clear idea of what to do: Calculate δn and δp from δf, neglecting phase-mixing terms (because they oscillate in phase space), expanding in small $\kappa v/\omega$, then work out from the relationship between δn and δp. No-one spelled out that the reason Landau damping was lost in the hydrodynamic equations was that the kinetic damping required all v

moments to be captured (although one student did show formally where in the calculation it was lost). The question has proved more challenging, on average, than I had anticipated.

Q3 was confined to material covered in the lectures or problem set and all candidates found it rather easy. The material is pretty advanced, however, and only published from 2012 onwards so I think the high scores are a tribute to the quality of the candidates. Several candidates lost marks through small algebraic errors or failure to answer a specific question probably through oversight rather than ignorance. The mark scheme proved satisfactory.

Nonequilibrium Statistical Physics

Q1 was a question on Master equation for some simple processes. It was attempted by 11 students whose marks ranged from 17 to 25. Q2 was on path integrals, and was mostly a repeat of coursework. It was attempted by 1 student who got full marks. Q3 was a variation on the Kramers and escape process. It was attempted by 12 students and marks were generally very high (in the range of 22-25). All solutions demonstrated a good understanding of the course material.

Quantum Field Theory

Q1: students proved very good understanding of this part of the syllabus. The only part which turned out to be problematic was a proper interpretation of the charge Q and of the commutation relations [Q,Phi]. Few candidates also struggled with a proper treatment of the function f while deriving equations of motion.

Q2: The main difficulty in this problem was in part c) where many students calculated the correlation function in momentum space instead of the correlation function in position space as it was asked in the question. Moreover, many candidates missed one or more Feynman diagrams in their answers. Most candidates proved to correctly read off Feynman integrals provided they found all Feynman diagrams.

Q3: The main difficulty was in part e): many candidates incorrectly set up a relevant integral as well as there were many computational mistakes in calculating the integral. Also, many students had difficulties with a proper derivation of Feynman rules.

Q4: Only few candidates decided to solve this problem. First three parts were solved by most of them correctly. However, they found difficult the last two parts. In particular, a proper reasoning to prove finiteness of the beta-function was missing in most cases.

Quantum Matter

There were no apparent problems on this exam. The marking was roughly as anticipated. For a few cases marks were given for answers slightly different from those in the official solutions if it was deemed the answer given was also an acceptable interpretation of the question. Question 2b in particular - it was not clear that one needed to write a final results as a sum over k. Instead any form was accepted.

Note, there were a few very high scores which brought up the average, an no scores that were extremely low to compensate. So I think the mean is artificially high.

Radiative Processes and High Energy Astrophysics

Small number of candidates, but all did very well.

Supersymmetry and Supergravity

The mean of the students' exam results was very high. I would like to note that at the end of the lecture course only a very motivated set of students was left who also performed well on the example sheets. Hence I am not surprised by this high average. Below are some small comments on the individual questions Q1: The overall standard was high. Most common (minor mistakes) were sign mistakes and discarding some terms too early. Q2: Again the quality answers was very high. I did not observe any common mistakes in this question. Q3: All candidates who chose to do this question had difficulties with part d of this question (determing gaugino and scalar masses). Apart from this sub-question, the standard was high. Q4: The standard of the answers was high in all sub-questions. The most deduction were in the part of matching matter multiplets of different supersymmetries.

C3.1: Algebraic Topology

Question 1. Very few candidates attempted this question even though it was probably the easiest on the exam, perhaps reflecting less comfort with algebraic than topological aspects of the course. There was one strong answer.

Question 2. Most candidates attempted this question, but many struggled with one or more sections of it. There was one strong answer. (b) Most candidates made reasonable attempts in this part, though there were some errors of detail and here and there a lack of clarity about the relative cohomology case. (c) For part (i) a number of candidates did a computation rather than simply applying Lefschetz duality. In part (iii), most but not all attempts to directly compute the simplicial cup product went awry, but there were a number of good answers using excision and identifying the quotient space and its cohomology ring.

Question 3. All candidates attempted this question, and the standard of answers was reasonable overall. There were three nearly perfect answers and a number of other decent ones. (a) A number of candidates either misidentified the boundary map in the Mayer–Vietoris theorem, or simply recapitulated the standard snake procedure for determining the boundary map without actually identifying explicitly the map in this case. (c) (i) Many candidates used a modified form of Mayer–Vietoris for the mapping torus (also obtainable as a long exact sequence of a pair), with two instead of four copies of the homology of K —this substantially reduced the complexity of the calculations. Nevertheless, in many cases despite obtaining the correct exact sequence, candidates made mistakes in computing the kernels and cokernels and piecing together the answer. (ii) There were many correct answers here, though also a few that mistakenly claimed the manifold was of dimension 2.

C3.4: Algebraic Geometry

Most candidates attempted questions 2 & 3. The quality of the scripts was exceptional, with almost flawless answers to every questions, even though the exam was about the same difficulty as in previous years.

The most common difficulty was question 3(d) with candidates not spotting an inverse map or running out of time.

Only one candidate had time to attempt three questions.

In question 2(b), R stands for the coordinate ring, this was announced during the exam, none of the candidates were confused in their answers to 2(b)

C5.6: Applied Complex Variables

1. This was by far the least popular question but was done well by many who attempted it. The basic conformal mapping was mostly handled competently, although few properly justified the final Möbius map to permute the points on the real axis. Those who could see how to use the hint generally managed the rather unpleasant calculations needed for part (c).

2. This was the most popular question. Few candidates convincingly performed the contour integration for part (b); many failed to use a consistent definition of the multifunction throughout. The bookwork in part (c) was mostly ok, though few candidates gave good explanations for the properties of H, in particular why it should be real on Γ.

3. This question was generally not well done. In part (a), most candidates seemed to understand the method but were let down by very many errors in basic manipulation (integration by parts, partial fractions, etc). Two candidates were misled by the hint inserted by the external examiner into expressing $g(\zeta)$ in terms of an unevaluated integral, making it harder to determine Γ. In part (b), most candidates were able to derive equation ($\star$), but the Wiener–Hopf decomposition caused many problems, particularly amongst those who were unable to identify the points $k = \pm 1$ in the Argand plane. Very few candidates completed the contour integration required for part (iv).

C3.2 Geometric Group Theory

Q1 This was a basic question about presentations and algorithmic problems. All students attempted this. Part a was done well. Many students had difficulties with part b.i. They did not realize that to produce a list they had to run two procedures 'in parallel' and that it is not possible to give a yes/no answer whether a given map extends to a homomorphism. Similarly in part b.ii some candidates did not realize that to check surjectivity it is enough to check whether the generators are in the image. Several did not explain properly how to check whether generators are in the image. In part b.iii most candidates did not realize that they had to search for appropriate homomorphisms $H' \to G$ rather that $H \to H'$.

Quite a few candidates who did not manage to do part b assumed the results and gave either complete or partial solutions to part c. Only 1 candidate managed to solve all parts of this question.

Q2 This was a question on amalgamated products and actions on trees attempted by most students. Part a was done generally well. In part b many students couldn't show that if a group has an infinite centre then a finite index subgroup has infinite centre too.

Part c.i was generally well done but some candidates failed to give a convincing proof as they tried to define the fixed point rather than simply show its existence.

Several students solved part c.ii using induction.

Few students did part c.iii. Very few realized that part c.ii was relevant and that one could use the action on the subtree of b.ii rather than T.

Q3 This question was attempted by only 2 students. It was on the last part of the course dealing with quasi-isometries and hyperbolic groups.

Both candidates did well in parts a and b gaining most points but they failed to tackle part c.

I think all question contained substantial pieces of bookwork and I see no excuse for the candidates that did poorly. Perhaps the students that are on the 2.i/2.ii border deserve a 2.i as candidates found questions 1b i,ii harder than I anticipated.

C3.3: Differentiable Manifolds

Question 1: Part (a) was bookwork, and well done. There were no serious attempts on parts (b),(c).

Question 2: This was the easiest question, and everyone attempted it. Only a minority could do (c)(ii).

Question 3: Not an easy question, but mostly well done by more able candidates.

C7.4: Introduction to Quantum Information

Question 1

This was the most popular question on the paper and generally well done. Parts (a) and (b) were bookwork and the solutions were mostly flawless. In part (c) some students did not notice that the measurement outcome $x = 0$ is inconclusive as there can be errors in both qubits. Most marks were lost in part (e) as many candidates struggled to describe the recovery procedure. Many good attempts at part (f).

Question 2

Fairly well done question. Very few students showed that $P_{\pm}$ satisfy conditions of orthogonal projectors. Many struggled with part (e). There were various attempts at part (f) but most students got the right idea.

Question 3

The bookwork in part (a) and the calculation of the Bloch vector in parts (c) and (d) caused no problems. However surprisingly many students struggled to prove positivity of a CPTP map in part (b). Common sloppiness and algebraic mistakes lead to mark losses in part (e).

C7.5: General Relativity I

Question 1

This exercise dealt with tensors in GR. While most students who attempted this exercise solved the first $\frac{2}{3}$, the unfamiliar nature of the last part caused problems to all of the students.

Question 2

Most students did well in this exercise, few were able to find $k(y)$ in (d). Happily, most had the idea for the correct parametrization in (c).

Question 3

This was attempted by the fewest students. While nearly everyone solved the introductory parts, the final part seemed too difficult for most.

C6.1: Numerical Linear Algebra

This seems to have been a reasonably successful exam with a range of marks including many high marks for the small number of undergraduate candidates.

C5.5: Perturbation Methods

Question 1. This was the least popular question. The bookwork in part (a) was very well done. In part (b) the application of Laplaces method was reasonably well done, though some candidates lost marks for failing to justify the size of the error term during each step of the argument or for failing to verify that the expansions are self consistent. In part (c) nearly all of the candidates identified the correct steepest descent contour, but only a handful dealt successfully with all of the steps required to derive the leading-order term in each regime.

Question 2. This was the second most popular question. In part (a) the bookwork on stating and applying Van Dykes Matching Rule was well done by all but a handful of candidates. In part (b) the application of the principle of dominant balance caused more problems than anticipated: many attempts did not consider all of the different cases in the expansion of $f(\epsilon^{\alpha}X;\epsilon)$ as $\epsilon \to 0^{+}$, though the minority that did then made efficient use of the hint. In part (c) the application of boundary layer theory caused even more problems, with many candidates failing to seek a boundary layer at both $x = 0$ and at $x = 1$ (the former despite observing in part (a) the non-uniformity in the expansion of $f(x;\epsilon)$ near $x = 0$).

Question 3. This was the most popular question. In part (a) the application of multiple scales theory was very well done on the whole. While many lost marks for algebraic slips in the derivation of the secularity condition or for failing to find the real part of $A(T)$ in the two cases, there were many excellent solutions. In part (b) the application of WKB theory to a third-order ordinary differential equation was well done by a significant minority, the majority receiving only partial credit.

C5.3: Statistical Mechanics

All the questions were done competently by the four MMathPhys students.

E. Comments on performance of identifiable individuals

Removed from public report

F. Names of members of the Board of Examiners

Examiners:

Prof Xenia de la Ossa
Prof John Chalker
Prof Andre Lukas (Chair)
Prof Gordon Ogilvie
Prof James Sparks
Prof Dan Waldram

Assessors:

Prof Steven Balbus
Prof Michael Barnes
Prof Charles Batty
Prof Tony Bell
Prof Simon Benjamin
Prof James Binney
Prof Stephen Blundell
Dr Andreas Braun
Prof Philip Candelas
Prof Joseph Conlon
Prof Garret Cotter
Prof Andrew Dancer
Dr Paul Dellar
Dr Amin Doostmohammadi
Prof Christopher Douglas
Prof Artur Ekert
Dr Christopher Eling
Prof Fabian Essler
Prof Andrew Fowler
Prof Ramin Golestanian
Prof Peter Grindrod
Dr Stephen Haben
Dr Ulrich Haisch
Prof Ben Hambly
Dr Heather Harrington
Prof Peter Howell

Prof Dominic Joyce
Dr Sven Krippendorf
Prof Ard Louis
Dr Tomasz Lukowski
Prof John Magorrian
Prof John March-Russell
Dr James Martin
Prof Lionel Mason
Dr Romain Mueller
Prof Jim Oliver
Dr Arijeet Pal
Prof Panos Papazoglou
Prof Felix Parra-Diaz
Prof Phillip Podsiadlowski
Prof Alexander Ritter
Prof Graham Ross
Prof Sakura Schafer-Nameki
Prof Alexander Schekochihin
Prof Graeme Segal
Dr David Seifert
Prof Steve Simon
Prof Andrei Starinets
Prof Caroline Terquem
Prof Ulrike Tillmann
Dr Eugene Vasilyev
Prof Vlakto Vedral
Dr Thorsten Wahl
Prof Andy Wathen
Prof Andrew Wells
Prof Julia Yeomans