A15809S1

Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

RADIATIVE PROCESSES AND HIGH-ENERGY ASTROPHYSICS TAKE HOME EXAM

Trinity Term 2020

TUESDAY 16TH JUNE 2020, 12 noon to THURSDAY 18TH JUNE 2020, 12 noon

You should submit answers to all questions.

You may write your answers by hand or type them. You may refer to books and other sources when completing the exam but should not discuss the exam with anyone else.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

1. Describe what is meant by the terms *forbidden transition*, *coronal approximation* and *critical density.* [3]

   Forbidden OIII lines are observed in the optical spectrum of a planetary nebula: one at a wavelength of 436.3 nm arising from transitions from the $^1S_0$ level down to the $^1D_2$ level, and another at 498.2 nm arising from transitions from the $^1D_2$ level down to the ^3P level. Considering OIII as a 3-level atom (we can treat the three sub-levels $^3P_0$, $^3P_1$ and $^3P_2$ as one level: ^3P), write down a general equation that balances electrons leaving and entering level 3, and another that balances electrons leaving and entering level 2. Modify these equations given that: 1) there are far fewer electrons in the $^1S_0$ level than in the $^1D_2$ level; 2) there are far fewer electrons in the $^1D_2$ level than in the ^3P level; and 3) the radiation field is weak.

   Given that the electron temperature in planetary nebulae is typically measured to be in the range $T_e \sim 8 \times 10^3 - 1.3 \times 10^4$ K, calculate the expected range of values for the critical density of decays from level 2. Given that the electron density n_e typically lies in the range $10^8 \leqslant n_e \leqslant 10^{10}$ m^{-3}, what process dominates decays in OIII ions in planetary nebulae? [8]

   At what electron temperature does the ratio $n_e C_{23}/A_{21}$ reach its peak value? Calculate the maximum value of this ratio for an electron density of $n_e = 10^{10}$ m^{-3}. What process therefore dominates transitions away from level 2? [5]

   Derive an expression for the flux ratio between the 436.3 nm and 498.2 nm emission lines in terms of the Einstein coefficients and line wavelengths. The observed line ratio is $F_{436.3\text{nm}}/F_{498.2\text{nm}} = 2.71 \times 10^{-3}$. Approximating the dust extinction curve as

   $$A_x = 4 + 3.3(x - 1.83) - 1.28(x - 1.83)^2,$$

   where $x = \mu\text{m}/\lambda$, calculate the temperature of the nebula. [9]

Transition	Wavelength (nm)	A (s^{-1})	Ω
$^3P - {}^1D_2$	498.2	0.0281	2.4
$^3P - {}^1S_0$	232.1	0.23	0.34
$^1D_2 - {}^1S_0$	436.3	1.6	0.31

[Note: the collisional de-excitation coefficient $C_{j,i} = 8.6 \times 10^{-12} \Omega_{j,i}/g_j \ (T_e/\text{K})^{-1/2} \text{m}^3\text{s}^{-1}$, where $\Omega_{j,i}$ is the collision strength and g_j is the statistical weight of level j.]

2. Describe the main similarities and differences between X-ray binary systems and active galactic nuclei (AGN). [4]

   A black hole of mass M is accreting via a thin disc (scale-height $h/r = 10^{-2}$) with a very large outer radius, and inner radius $r_{\rm in} = 6r_g$, where $r_g = GM/c^2$. The accretion luminosity is $L = 0.5L_{\rm Edd}$, where $L_{\rm Edd} = 1.26 \times 10^{31} M/M_\odot$ W is the Eddington luminosity. Calculate the efficiency of the accretion disc, and calculate the peak disc temperature for two values of black hole mass: $M = 10\ M_\odot$ and $M = 10^8\ M_\odot$. Comment on what your results mean for the spectrum of the accretion disc in X-ray binaries and AGN. [5]

   Derive an expression for the disc density as a function of distance above the mid-plane, $\rho(z)$. Given that the disc is radiation pressure dominated ($P = 4\sigma T^4/3c$), calculate $\rho(z = 0)$ at $r = r_{\rm in}$ in units of kg cm^{-3} for the same two values of black hole mass. [5]

   A fraction f of the accretion luminosity powers a hot X-ray corona, which irradiates the disc leading to the emission of an iron Kα emission line. The centroid frequency and width of the line in the disc restframe depend on the ionisation parameter $\xi \propto F_x/\rho(z = H)$, where F_x is the irradiating flux from the corona that is crossing the disc surface. Given that $\xi(r = r_{\rm in}) \propto M^\beta$, calculate β. [3]

   Iron line photons emitted from a given rotating disc element with a frequency of ν' in the local rest frame are measured to have a frequency $\nu = \nu'\ \delta$ by the observer, where δ is the Doppler factor. According to the observer, the disc element is moving with a velocity v at an angle θ to the line of sight. According to the disc element, this angle is instead θ', where $\cos\theta' = [\cos\theta - v/c]/[1 - (v/c)\cos\theta]$ in Special Relativity. Given that an interval in solid angle in the disc element rest frame $d\Omega'$ transforms to the observer's rest frame as $d\Omega = d\Omega'/\delta^2$, derive an expression for the Doppler factor. Show that for a flat Keplerian disc with inclination angle i, the maximum Doppler factor for a disc element at radius r is

   $$\delta_{\rm max} = \frac{\sqrt{1 - r_g/r}}{1 - (r/r_g)^{-1/2}\sin i}.$$

   [5]

   What two effects change this expression when General Relativity is fully accounted for? Comment on what your values of β and $\delta_{\rm max}$ mean for the iron line observed from X-ray binaries and AGN. Why does the above reasoning break down for $L = 10^{-5}L_{\rm Edd}$? [3]

   $\sigma = 5.6704 \times 10^{-8}$ W m^{-2} K^{-4}, $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-1}, $M_\odot = 1.99 \times 10^{30}$ kg.