Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

CRITICAL PHENOMENA
TAKE-HOME EXAM

TRINITY TERM 2016

MONDAY, 20 JUNE 2016, 12noon to WEDNESDAY, 22 JUNE 2016, 12noon

You should submit answers to **all** *questions. Answer booklets are provided for you to use but you may type your answers if you wish. Typed answers should be printed single-sided and the pages securely fastened together.*

You may refer to books and other sources when completing the exam but should not discuss the exam with anyone else.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

1. [10 marks] Use the mean-field theory computation of $G(r) = \langle(\phi(\vec{r}))\phi(0)\rangle$ discussed in class or in chapter 2 of Cardy's book to find ξ_0 and the critical exponent ν, defined by writing the correlation length as

$$\xi \sim \xi_0 |t|^{-\nu}$$

where $t = (T - T_c)/T_c$. Do this both for $T > T_c$ and $T < T_c$, and find the ratio of the amplitudes ξ_0 in the two cases.

2. [10 marks] Compute the scaling dimension of the operator $\cos(p\phi)$ in the Gaussian model in two dimensions. You don't have to go through every detail of defining the sum over all states, but you must at least discuss the important issues. You must then do the calculation, and indicate which regulator you use. Make sure you define the stiffness precisely, and then find the values of stiffness for which this operator is relevant.

3. [10 marks] The purpose of this problem is to study the Ising model on the square lattice using the Migdal-Kadanoff recursion procedure. This is an alternative to decimation, consisting of the two steps illustrated in the figure below. First one removes half the bonds, but leaves all the sites. To compensate for this, each of the remaining bond strengths is doubled to $2J$. The second step is to then sum over (i.e. decimate) the spins with only two neighbors (the big circles in the middle figure), giving the square lattice on the right.

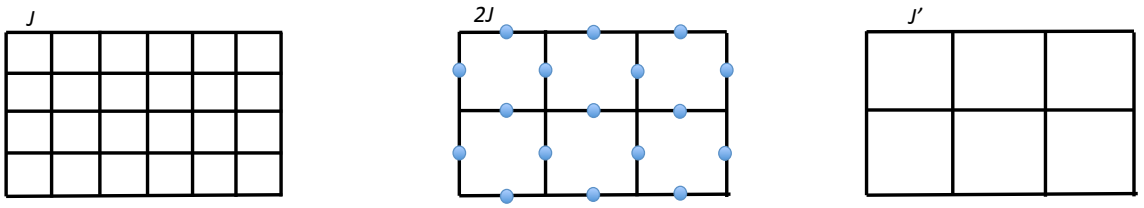
   (a) What is the scale factor b?

   (b) Find the recursion relation for the square lattice, using the variable $x = e^{2J/T}$.

   (c) Find x^*, the value of the coupling at the non-trivial fixed point, and compare it to the exact value.

   (d) Linearize around the fixed point and find the eigenvalue of the RG transformation. Is it relevant?

   (e) Find the exponent ν, as defined in problem 1, in this approximation.



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