Examiners' Report: Final Honour School of Mathematical and Theoretical Physics Part C and MSc in Mathematical and Theoretical Physics Trinity Term 2024

October 31, 2024

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

	Numbers							
	2024	2023	2022	2021	2020	2019	2018	2017
Distinction	53	37	38	42	42	40	25	31
Merit	13	7	13	10	9	6	n/a	n/a
Pass	14	9	11	12	3	6	17	10
Fail	6	4	1	3	1	1	0	0
	Percentages %							
	2024	2023	2022	2021	2020	2019	2018	2017
Distinction	62	65	60	63	76	76	60	76
Merit	15	12	20	15	17	11	n/a	n/a
Pass	16	16	17	18	5	11	40	24
Fail	7	7	2	4	2	0	0	0

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted.

- **Marking of scripts.**
  All dissertations and two mini-project subjects were double-marked. In cases of significant disagreement between marks, the two markers were consulted to agree a reconciled mark.

  All written examinations and take-home exams were single-marked according to checked model solutions and a pre-defined marking scheme. A comprehensive independent checking procedure was followed.

B. New examining methods and procedures

None.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

Notices to candidates were sent on: 16th October 2023 (first notice), 28th November 2023 (second notice), 9th April 2023 (third notice), and 24th May 2024 (fourth notice).

The examination conventions for the 2023-2024 academic year are online at http://mmathphys.physics.ox.ac.uk/students.

Part II

A. General Comments on the Examination

Removed from public version

B. Breakdown of the results by gender

Removed from public version

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 2 and in the Average USM per Formal Assessment graph below. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Paper	Number of Candidates	Avg USM	StDev USM
Advanced Fluid Dynamics	6	65	19.8
Advanced Philosophy of Physics	-	-	-
Advanced Quantum Field Theory	56	70	16.5
Advanced Quantum Theory	27	72	14.0
Algebraic Geometry	-	-	-
Algebraic Topology	2	-	-
Analytic Number Theory	1	-	-
Applied Complex Variables	7	69	13.7
Collisionless Plasma Physics	9	64	20.4
Differentiable Manifolds	13	65	20.1
Dissertation (single unit)	19	76	-
Dissertation (double unit)	34	80	-
General Relativity I	51	64	13.6
General Relativity II	34	63	16.2
Geometric Group Theory	2	-	-
Geophysical Fluid Dynamics	1	-	-
Groups and Representations	61	80	15.17
Introduction to Quantum Information	37	72	17.3
Kinetic Theory	5	61	22.4
Low-Dimensional Topopology and Knot Theory	-	-	-
Networks	9	69	6.7
Numerical Linear Algebra	5	72	13.4
Perturbation Methods	21	58	11.8
Quantum Field Theory	79	67	15.0
Radiative Processes and High Eng. Astro.	2	-	-
Random Matrix Theory	15	64	15.3
Riemannian Geometry	4	-	-
String Theory I	45	73	4.5

Table 2: Statistics for individual papers

The number of candidates taking each homework-completion course is shown in Table 3. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Paper	Number of Candidates	Percentage completing course
Advanced Fluid Dynamics	1	-
Advanced Philosophy of Physics	3	-
Advanced Topics in Plasma Physics	-	-
An Introduction to Topological Phases of Matter	14	100
Collisionless Plasma Physics	-	-
Collisional Plasma Physics	1	-
Conformal Field Theory	32	100
Cosmology	23	100
Galactic and Planetary Dynamics	8	87.5
Group and Representations	60	98.3
High Energy Density Plasma Physics	2	-
Kinetic Theory	9	100
Nonequilibrium Statistical Physics	8	100
Quantum Field Theory in Curved Space	30	100
Quantum Matter	18	100
Quantum Processes in Hot Plasma	4	-
Renormalisation Group	8	100
Statistical Mechanics and Computer Algorithms	23	100
String Theory II	8	100
Supersymmetry and Supergravity	26	100
The Standard Model and Beyond I	13	92.3
The Standard Model and Beyond II	8	100
Topics in Soft and Accretion Matter Physics	-	-
Topological Quantum Theory	-	-

Table 3: Numbers taking each homework completion course

D. Assessors' comments on sections and on individual questions

Advanced Fluid Dynamics

Question 1.

Most students performed at Merit or Distinction level on this question.

All students scored perfect marks on the simplest part (a).

Parts (b) and (c) proved more difficult than expected, particularly part (b) which was a simple derivation of a classic result from dynamics. Most candidates failed to notice that moving into a frame rotating with a fluid results in the fluid being at rest in that frame. This lead to numerous attempts which would have been correct had $\vec{u} = \vec{0}$ been applied. This mistake propagated into the more difficult part (c). The key to part (c) was to note that force balance (part b) requires $\vec{\nabla}\Phi = \Omega^2(R)R\hat{R}$, which can be used in the displaced force balance equation to relate all quantities to the rate of rotation $\Omega(R)$

$$|\Delta f| = |f_{\text{gravity}} - f_{\text{rotation}}| = \rho|\Omega^2(R + \xi_R)(R + \xi_R) - \Omega^2(R)(R + \xi_R)|$$

which leads to the desired result upon expansion.

Parts (d) and (e) were well answered by those candidates who made attempts. Part (d) was covered in the lectures and was perfectly answered by nearly all candidates. Part (e) proved more difficult, although all candidates seemed clear on the required approach. Errors in part (b) resurfaced, and the lack of zero mean flow $\vec{u} = \vec{0}$ hampered many candidates attempts. Candidates clearly knew how to derive dispersion relationships, and the criteria which must be satisfied for stability (or lack thereof). Most mistakes in this final section were of a technical nature, and the physical intuition of the candidates appeared to be correct.

Question 2.

Most candidates performed at merit level or better on this question. Almost all candidates achieved full marks for parts (a) and (b). It is easiest to show that the viscous dissipation in a Newtonian fluid is non-negative by writing $\Phi = 2\mu\mathsf{e} : \mathsf{e}$ using the symmetric strain rate e. The velocity gradient $\nabla\mathbf{u}$ is stated to be uniform at the start of the question, so there is no need for a Taylor expansion in part (b).

Part (c) caused more difficulty than expected. The viscous dissipation due to the motion of a rigid particle in Stokes flow, like the drag force, is a function of the particle's velocity *relative to the fluid around it*. More precisely, it is minus the dot product between the Stokes drag force on the particle and the velocity of the particle relative to the surrounding fluid many diameters from the particle. This leads to the given expression proportional to $|\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}|^2$.

One can see this by applying the result in part (a) to the Stokes flow outside a rigid particle. The left-hand side vanishes as the $\mathrm{d}/\mathrm{d}t$ term is negligible. The right-hand side becomes invariant under Galilean transformations as $\nabla \cdot \boldsymbol{\sigma} = 0$ in the fluid. The viscous dissipation in the fluid outside the particle is then equal to an integral over the surface of the particle plus an integral over a surface at infinity. The latter vanishes in the frame in which the fluid velocity many diameters from the particle tends to zero. This argument assumes that the background velocity is approximately uniform over many particle diameters, or equivalently finds the dissipation due to the motion of the particle, rather than the dissipation due to $\nabla\mathbf{u}$. The integrated viscous dissipation for each particle is then $\phi_i = \zeta|\dot{\mathbf{r}}_i - \mathbf{r}_i \cdot \nabla\mathbf{u}|^2$, and their sum is $\phi_1 + \phi_2 = \frac{1}{2}\zeta|\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}|^2$ as in the question.

For part (d), the expected approach uses the given equation ($\star$) for $\dot{\mathbf{R}}$ once to write

$$\frac{1}{2}\zeta\left|\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}\right|^2 = -(\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}) \cdot (H\mathbf{R} + kT\nabla_{\mathbf{R}} \log \psi).$$

This contains one instance of $\nabla_{\mathbf{R}} \log \psi = \nabla_{\mathbf{R}} \log \Psi$ so applying $\langle\langle \cdots \rangle\rangle$ gives an expression that is linear in Ψ,

$$\left\langle\!\!\left\langle \frac{1}{2}\zeta\left|\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}\right|^2 \right\rangle\!\!\right\rangle = -\iint (\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}) \cdot (H\mathbf{R}\Psi + kT\nabla_{\mathbf{R}}\Psi)\mathrm{d}\dot{\mathbf{R}}\mathrm{d}\mathbf{R}.$$

The contribution proportional to $\dot{\mathbf{R}}$ vanishes because the distribution $\Psi(\mathbf{R}, \dot{\mathbf{R}}, t)$ is symmetrical about $\dot{\mathbf{R}} = 0$, leaving

$$\begin{aligned}\left\langle\!\!\left\langle \frac{1}{2}\zeta\left|\dot{\mathbf{R}} - \mathbf{R} \cdot \nabla\mathbf{u}\right|^2 \right\rangle\!\!\right\rangle &= nH \int R_i R_j \partial u_j/\partial x_i \psi \mathrm{d}\mathbf{R} + nkT \int R_i \partial u_j/\partial x_i \partial\psi/\partial R_j \mathrm{d}\mathbf{R}, \\ &= nHC_{ji}\partial u_j/\partial x_i - nkT \int \partial u_j/\partial x_i \partial R_i/\partial R_j \psi \mathrm{d}\mathbf{R} = nH\mathsf{C} : \nabla\mathbf{u},\end{aligned}$$

as C is symmetric, and $(\partial u_j/\partial x_i)(\partial R_i/\partial R_j) = (\partial u_j/\partial x_i)\delta_{ij} = \nabla \cdot \mathbf{u} = 0$ in an incompressible fluid.

Most candidates did not use ($\star$) at all, and so arrived at a more complicated expression involving C that would simply using the evolution equation for C in part (b). This approach received substantial partial marks.

The viscous dissipation is equal to $\sigma : \nabla\mathbf{u}$, where $\sigma = nH\mathsf{C}$ is the stress due to the bead-spring pairs as seen in lectures. This matches the general result derived in part (a).

Advanced Quantum Field Theory

The exam average grade was 51.8 out of 75. Most students made substantive progress on all 3 questions.

Question 1 was on the Dirac Lagrangian. The average mark was 15.9 out of 25. A number of students were sloppy on the logic of the derivation of Noether's theorem. Very few students correctly identified the symmetries in the last parts of the question, and the appropriate Noether currents for them.

Question 2 was on Quantum Electrodynamics. This was a high scoring question with an average of 18.2 out of 25. A few students stumbled at the ordering within the trace in the Feynman rules, and a few others did not use the hint in part (c), carrying out a longer calculation. Many students did not give a clean argument for the finiteness of the 4-point amplitude.

Question 3 was on non-Abelian gauge theory. The mean mark was 17.7 out of 25. Most students were able to write down the diagrams and amplitudes correctly. The diagrams were sometimes incompletely labeled, leading to errors in the amplitude. A few students got lost in algebra to prove (d).

Advanced Quantum Theory

Question 1. This question concerns the imaginary time propagator for the quantum harmonic oscillator. It was done moderately well (10/27 answers at or above the default Distinction level, and 6/27 answers below the default Pass level). The best answers were perfect or near-perfect. Weak candidates were unable to derive in a convincing way the expression for the action of the saddle-point path that was given in the question, and a few could not derive the correct Euler-Lagrange equation for the saddle-point path, although even these were able to pick up a few marks later in the question.

Question 2. This question concerns the Bogoliubov theory for a weakly interacting gas of bosons at zero temperature. It was done quite well (19/27 answers at or above the default Distinction level, and 2/27 answers at the default Pass level; none lower than that). Almost all candidates had a reasonable understanding of the essentials but weaker candidates could not derive the expression given in the question for the expectation value of the Hamiltonian in the trial state. No candidates really explained why correlations between bosons in the Bogoliubov state lower this expectation value.

Collisionless Plasma Physics

Q1 and Q2 examined the HT part of the course that deals with plasma waves. On average, students got about 2/3 of the points in both questions, with only one outlier who got only 30%. With one exception, the points awarded for these two questions correlate quite well for each individual student, suggesting that both problems were of comparable difficulty.

The first two parts of Q1 were bookwork; still, most students managed to lose a mark or two by failing to point out obvious approximations that they are making or the fact that Ω_e is defined to be positive by convention (as stated during lectures). Most students realised (correctly) that solving Q1(e) requires solving a quartic polynomial, whose solutions were essentially given (the cutoffs and resonance points mentioned in the problem statement). I was surprised to see that, in Q1(f), nobody noticed that, for non-electrostatic waves, the cold-plasma dispersion relation is well defined only if n^2 is a real number (this is shown in the lectures and is in the lecture notes). Thus, k^2 must be real for the WKB scheme as discussed in the lectures to make sense. Q1(g) required the students to reason physically about what's going to happen in a situation that was new to them (or at least was not discussed in the lectures). It was nice to see that nearly half of the candidates figured it out.

Nobody figured out the answer to the last question posed in Q2(b), viz., (14) is not valid near the cutoff points as the approximations under which it is derived break down; (7)–(9) are still valid for any ξ (away from a resonance). A few students made simple errors/typos in (c) and (d) which were certainly the easier parts of Q2.

Q3 was on the longish side in terms of formal calculations, but those were mostly of standard flavour and could have been easily generalised from the lecture notes, which, as well as ample time to peruse them, were availble with this take-home exam. The candidates' marks covered a broad range, providing clear proof of principle that it was within an MTP student's abilities to achieve near perfection in this exam, but it was also within his/her abilities to achieve very little indeed. It seems, therefore, that the question worked well as an exam question.

(a) A surprising number of candidates did not realise that this was a prompt to use the electron drift-kinetic equation to derive the Boltzmann response for the electrons and then use quasineutrality to relate φ and δf_i: several of them just wrote the perturbed ion equation and derived the linear relationship between δf_i and φ. I did not penalise them for that as perhaps there was indeed ambiguity in the phrasing of the question as to what was required—although those who did not think of quasineutrality or electrons at all found themselves in some difficulty later on as they could not close the equations and derive a proper dispersion relation.

(b) All knew to take moments and most managed to do it successfully (although some discovered that care—and some adjustments—were required in copying results from lecture notes: e.g., when differentiating f_0 with respect to $v_\parallel$, one had to remember that the Maxwellian was now with respect to $v_\parallel - u_0$).

(c) Fourier-transforming and finding a quadratic dispersion relation from two coupled linear PDEs proved firmly within our students' abilities, except for those who did not realise that quasineutrality and Boltzmann electrons were required to relate δn_i to φ. No one spotted that the PVG mode was a destablised sound wave, but they mostly could see how the instability loop worked.

(d) This was a longish calculation, but exactly analogous to what was done in the lecture notes. A fair number of candidates saw that and did it well.

(e) Most understood that the fluid limit meant $\zeta \gg 1$, but not all of them realised that $Z(\zeta)$ needed expanding to two leading orders. Those who did, successfully recovered the fluid result, and most of them also understood why pressure was negligible (δf_i was approximately odd in $v_\| - u_0$ in this limit) and why the fluid limit took one correctly all the way to the maximum growth rate only for cold ions ($\zeta \sim \bar{\tau}^{-1/2}$ at this maximum).

(f) Again, most knew how to get the stability boundary (assume real ζ), and most of those managed to derive the required formula from that (some showed 21st-century acumen and used Mathematica, which is fine by me).

(g) Getting to the required condition of instabilty from the result of (f) was not much of a challenge (just making the argument of the square root positive), but the sketching of growth rates proved the most difficult part of the question. Few understood clearly that one had to classify all cases into $\eta_i > 2$ and $\eta_i < 2$ as well as $\eta_u > 0$ and $\eta_u < 0$, and even fewer worked out what the sketches should look like in each case. This was somewhat surprising as plotting the solutions via Mathematica (or equivalent) could have given them the answers to rationalise.

Geophysical Fluid Dynamics

Q1. This was a popular question which was done reasonably well; the average mark was pulled down by a few very brief attempts. Overall, most marks were lost by not fully attempting the interpretation parts of the question. Other common mistakes were not considering or discussing the +/- options for omega in part b, and failing to find that u is proportional to h in part c. Part d was generally done very well, and several students made reasonable points in part e and got 1-2 marks.

Groups and Representations

A few exceptions aside, students completed this paper to a very high standard and demonstrated that they have absorbed the relevant material.

Question 1 This was a question on the finite group S3 and its representations and a final application to a scalar field potential. It was attempted by all students and with an average performance of 20.0 marks. Most problems arose in the final part of the question where some students embarked on long calculations when they should have simply applied the results from the previous parts.

Question 2 This question was on the group SU(4) and its properties with a final application to a quark model of mesons. It was attempted by all students with an average of 19.7 marks. A common problem in part b) was that some students simply used Dynkin formalism rather than calculate the weights of the standard unit vectors, as instructed.

Question 3 A very unpopular question, properly attempted by only one student.

Question 4 A question on unitary groups and SU(6) in particular, with a final application to a grand unified model. This was attempted by all students with an average mark of 20.5. Some problems arose in part b), mainly because arguments for surjectivity of the map and the form of its kernel were not conclusive.

Kinetic Theory

Question 1

Most candidates obtained full marks on parts (a) and (b). A few candidates derived the full BBGKY hierarchy rather than the slightly simpler special case with $s = 1$ that was asked for. Both approaches received full marks. The only real issue was remembering that $\partial\phi(|\mathbf{x}_1 - \mathbf{x}_j|)/\partial\mathbf{x}_i$ for $j \neq 2$ gives two nonzero terms, one with $i = 1$ and one with $i = j$. The latter becomes part of an exact divergence.

Most candidates completed the first part of (c), but made little further progress. It is best approached by *verifying* the given expression for the pressure tensor. Calculating its divergence with the aid of the first result $\partial n_2/\partial s = \mathbf{R} \cdot \partial n_2/\partial \mathbf{x}$ gives

$$\begin{aligned} \frac{\partial}{\partial x_i}\mathsf{P}^{\phi}_{ij} &= -\frac{1}{2}\int \mathrm{d}\mathbf{R}\,\frac{R_j}{R}\frac{\mathrm{d}\phi}{\mathrm{d}R}\int_0^1 \mathrm{d}s\,\frac{\partial}{\partial s} n_2(\mathbf{x} + (s-1)\mathbf{R}, \mathbf{x} + s\mathbf{R}, t), \\ &= -\int \mathrm{d}\mathbf{R}\,\frac{R_j}{R}\frac{\mathrm{d}\phi}{\mathrm{d}R}\, n_2(\mathbf{x}, \mathbf{x} + \mathbf{R}, t), \end{aligned}$$

using the symmetry $n_2(\mathbf{x}, \mathbf{x}_s, t) = n_2(\mathbf{x}_s, \mathbf{x}, t)$ for indistinguishable particles. Finally, put $\mathbf{x}_s = \mathbf{x} + \mathbf{R}$ and use the chain rule for $\mathrm{d}\phi/\mathrm{d}R$ with $R = |\mathbf{x}_s - \mathbf{x}|$.

Only one candidate completed part (d). The system is spatially uniform and isotropic, so ρ is spatially uniform, $n_2(\mathbf{x}, \mathbf{x}_s, t) = \rho^2 g_2(R)$ is spherically symmetric, and the fluid velocity $\mathbf{u} = 0$. The tensor $\mathsf{P}^{\phi}_{ij} = p^{\phi}\delta_{ij}$ is isotropic. Taking the trace of the given expression for P^{ϕ}_{ij} determines p_{ϕ}.

The pressure p is larger than $\rho\theta$ for a repulsive potential. The integral is negative as $g_2(R) \geq 0$ by definition. The trace of the pressure tensor differs from $\rho\theta$ because this kinetic equation is not the Boltzmann equation. The right-hand side of the evolution equation for f in (a) does not conserve the 1-particle kinetic energy.

Question 3

There was a broader range of marks on this question compared to previous years (from 10/25 to 22/25) despite only five students taking the exam. The average mark was 17 / 25, which suggests the question was set at about the right level of difficulty.

Part (a) — everyone knew why angle-action variables are important, but a few people failed to write down the collisionless Boltzmann equation correctly, losing an easy mark. Strangely this did not affect anyone's answer to part (b).

Part (b) — all done perfectly.

Part (c) — essentially done perfectly by those who attempted it.

Part (d) — largely done very well (marks ranging from 6/9 to 9/9). Those who dropped marks usually did so because they (i) did not give a convincing answer about analytic continuation via Landau's prescription (sometimes confusing this with the inverse Laplace transform) or (ii) did not interpret zeroes of ϵ as Landau modes (I also accepted 'eigenmodes', 'growing/decaying modes', etc.), or both (i) & (ii).

Part (e) — much less well done. Probably the easiest solution is to write down the density of the particle as a delta function in real space and then Fourier transform and multiply by $-4\pi G/k^2$. This was done perfectly by one candidate. Others clearly had the right idea but struggled with the details.

Parf (f) — variable answers, much like in part (e). Some candidates seemed to think that a lot of mathematical manipulations were required, but in truth one just needs to note that the external potential has a pole at $\omega = \mathbf{k} \cdot \mathbf{v}_\mathrm{p}$, from which the answer follows in a line or two. Nobody interpreted the final result particularly clearly. On the other hand, it was gratifying that several students knew this result was valid after only a time $t \gg |\mathrm{Im}\,\omega|^{-1}$, something often misunderstood in the literature!

Quantum Field Theory

The paper was generally well done although candidates often lost marks by lazy imprecise algebra, and some failed to address the question actually asked.

1. Q1 average mark 16.1. Some candidates did not manage to make the transition from the free field assumed in the first three parts to the general interacting field of the last two parts. Finding the state $|\psi\rangle$ in part e) defeated all but the strongest candidates.

2. Q2 average mark 16.9. Some candidates tried to prove the properties of the projection operator in part b) by using memorised explicit representations of spinors which is not what was asked. Only a few candidates realised that the crossing symmetry in fermion-antifermion scattering could be used to restrict the form of the interference term between t and s channel graphs in part d).

3. Q3 average mark 15.7. The first two parts were bookwork and very well done. In part c) many candidates produce a great deal of superfluous information by trying to analyse individual divergent graphs – they were asked simply to list the superficially divergent correlation functions. The last part was poorly done.

Radiative Processes and High Energy Astrophysics

The following comments reflect the performance of all students who took the full C1 paper and the shorter paper for the MMathPhys students which was only these three questions

Q1. The majority of students answered this well. (a) and (b) were fairly standard recall questions from taught material and the broad idea of shock acceleration was demonstrated by most. In (c) the aspect of Roche Lobe overflow was understood, and the potential for stellar wind accretion was mentioned by a minority. Students found (d) and (e) quite straightforward but the simple calculation following in (f) was surprisingly not completed by many.

Q2. Students tended to lose marks across all of parts (a) to (e). A significant number didn't pause for thought to sketch the function correctly and consider what they were plotting, which should have been straightforward. But the integral in (a) was comfortable for virtually all who answered this question. Most students produced the solutions for (b) and (c), although dropping of some terms or incorrect definitions of the terms was relatively common. Like part (a), some time and consideration would have caught most errors. A large number found (d) more challenging. Although most got the equation setup with the condition for hydrostatic equilibrium and the solution for M(r), the two integrals proved challenging to complete correctly. The next step of getting the T dependence was done well for those that made it through the solution for P(r). And again for (e), only those that completed the previous two parts scored well.

Q3. By some way this was the least popular question. Parts (a) and (b) were relatively straightforward for all. In part (c) the major difficulty was just in defining the number density of H and He. For all who managed that, then the f_v expression was then achieved. (d) and (e) were somewhat easier than many thought, and are actually more of a definition rather than derivations. (f) was challenging and required several steps and only two students of the 6 answers managed to make significant headway.

C3.1: Algebraic Topology

Some students erroneously answered that there was only one 0-cell in Question 1(a)(i), which has the effect of making subsequent computations significantly easier. Most students failed to recognise X in Question 1 as a torus. This make it particularly challenging to answer Question 1(c)(i).

Question 2 was done well by most students, with the exception of (b)(iii) which proved particularly tricky.

Question 3 was by far the most challenging, and the fact that the figure was missing made it essentially impossible to do.

C3.2: Geometric Group Theory

Question 1. This question was the most popular. Some candidates missed the requirement that A′, B′,C′ should be pairwise distinct. Part (b) was well answered and displayed a sound knowledge of normal forms and structural properties of amalgamated products and their subgroups.

Question 2. This question was quite popular as well. Most candidates were able to produce an accurate construction of a free basis for a group acting freely on a tree. Part (b) was well answered too, with several smart answers, but also a few answers where some confusion between free groups and their bases, and linear spaces and their bases was perceptible.

Question 3. This question was attempted by less than half of the candidates, and those who answered it did not obtain high marks.In the end of (a), (ii), almost all candidates missed the fact that there were two cases to discuss. Parts (b) and (c), (ii), were answered well, question (c) (i) less so, even if it was very close to arguments seen.

C3.3: Differentiable Manifolds

Question 1. For (c), the answer I hoped for was along the lines of

$$\begin{aligned}
\mathcal{L}_v(i_w(\alpha)) &= \frac{\mathrm{d}}{\mathrm{d}t}\big(\varphi_t^*(i_w(\alpha))\big)\Big|_{t=0} = \frac{\mathrm{d}}{\mathrm{d}t}\big(i_{\varphi_t^*(w)}(\varphi_t^*(\alpha))\big)\Big|_{t=0} \\
&= \Big(i_{\frac{\mathrm{d}}{\mathrm{d}t}\varphi_t^*(w)}(\varphi_t^*(\alpha)) + i_{\varphi_t^*(w)}(\frac{\mathrm{d}}{\mathrm{d}t}\varphi_t^*(\alpha))\Big)\Big|_{t=0} \\
&= i_{\mathcal{L}_v w}(\varphi_0^*(\alpha)) + i_{\varphi_0^*(w)}(\mathcal{L}_v\alpha) = i_{[v,w]}(\alpha) + i_w(\mathcal{L}_v\alpha),
\end{aligned}$$

but few did this correctly. In (d), combining (c) and Cartan's formula gives an equation involving $+i_w \circ i_v(\mathrm{d}\alpha)$ rather than $-i_v \circ i_w(\mathrm{d}\alpha)$, and only minority noticed this and explained why they are equal. For (e), one should prove the result by induction on increasing k, which was often not completed.

Question 2. Part (c) was modelled on the computation of $H^*(\mathcal{S}^n)$ in the classes, but many candidates could not get to the end. In (c)(ii), the correct answer was $\epsilon = \eta_U\beta + \eta_V\gamma + \delta \wedge \mathrm{d}\eta_U$. Very few candidates got this, most wrote $\epsilon = \eta_U\beta + \eta_V\gamma$ and failed to notice that $\mathrm{d}\epsilon \neq \alpha$.

Question 3. The raw marks on this question varied between 0 and 25. Candidates who understood the material scored highly, others did poorly. Some candidates did not understand part (b) and attempted to answer (b)(i)–(iv) independently, rather than finding functions d_i satisfying all of (i)–(iv).

C3.8: Analytic Number Theory

Overall this years question was reasonably successful, producing a good spread of marks with all questions reasonably popular amongst candidates (question 3 was slightly less popular). A few candidates seemed to have suffered from time issues (particularly with question 3), but this did not appear to be too widespread. The questions succeeded in distinguishing between candidates, and almost all candidates were able to demonstrate at least a basic understanding of core concepts in the course. In question 1 the bookwork part (a) was almost universally answered correctly. In part (b) many candidates struggled with demonstrating the inverse, which was not covered in lectures and so understandably harder. It was slightly surprising that despite many similar questions in examples sheets and revision sessions, a number of candidates didn't know the standard strategy for attempting (c).

Question 2 had the lowest average mark from candidates who attempted it. The most challenging part (c) did a good job of distinguishing between candidates, since at some points it required less obvious modifications of content covered in lectures.

Question 3 was the least popular question, but answered well by most candidates who attempted it. In hindsight the question was maybe slightly too long for the final question in an exam; several candidates didn't attempt part (d) even if this was less difficult in general. Several candidates didn't spot the relevance of (b)(i) to (b)(ii) despite the language used, which caused them to slip up slightly. It was pleasing that most candidates understood the overall strategy pretty well, and the technical execution of this distinguished between different candidates pretty well.

C3.11: Riemannian Geometry

Question 1. Part (a) was done well. Part (b)(i) was usually done well, though some students decided to use an incorrect statement of the Koszul formula which led to difficulties. In part (b)(ii), students had the right idea but often lost marks for lack of justification (e.g. that the V_i are orthogonal). Part (c) proved challenging with computational errors and lack of justification. This was a popular question done by almost all students.

Question 2. Part (a) was done well. Part (b)(i) was done well, with the common error being not to note that the required distance is bounded above by $L - t$. Part (b)(ii) was done quite well, with marks typically lost for not recognising the significance of $|\alpha'| = 1$ (i.e. to get the correct parameterisation). Part (b)(iii) proved challenging, with students unable to construct a suitable variation. Part (c) elicited few attempts, but they were done well. About half the students attempted this question.

Question 3. Part (a) was done well by almost all students. Part (b) had a mixed response. As expected, the usual error in (b)(i) was to assume the metric was complete, whereas others correctly spotted the connection to hyperbolic space. Part (b)(ii) was usually done well. Part (b)(iii) had similar issues to (b)(i) again as expected, but again a good number of students spotted the connection to the round metric in higher dimensions. Part (b)(iv) was usually done well, with the most common error just to miss justification of completeness or that the product metric constructed as non-negative curvature. Part (b)(v) was also usually done well, with the only error begin lack of justification of a uniform lower bound on Ricci curvature. This was another popular question attempted by most students.

Summary. The exam went well with a good range of marks. Question 2 appeared to be a bit more challenging for the students, but not overly different from Questions 1 and 3 which seemed to be roughly equal difficulty.

C3.12: Low-Dimensional Topology and Knot Theory

Question 1 (7 attempts): This question tested knowledge of handle decom-positions of smooth manifolds, especially surfaces, and their relationship to homology. The general level of solutions was high. In (b)(i), some candi-dates did not realise that the coefficients of the boundary map are given by the algebraic intersection number between the attaching circle and the belt circle. In (b)(ii), some candidates missed the half twist in the 1-handle. There were no complete solutions for (b)(iii), though several solutions made good progress.

Question 2 (7 attempts): This question tested knowledge of the linking num¬ber, Seifert form, and their relation to the intersection form of the Seifert surface. The general level of solutions was good. In (a)(i), some candidates forgot to say that one only considers crossings between K and K' formula for the linking number. In (a)(ii), some did not consider the oriented smoothing, hence potentially ending up with a non-orientable surface. Unfortunately, there is an absolute value missing from the definition of the determinant in the statement of part (b), which makes the determinant well-defined only up to sign in (b)(i). No marks were deducted in (b)(i) related to any sign issues, and most candidates noticed the problem. There was no complete answer to (b)(ii), though there was one essentially complete solution. Some candidates considered projections of curves on the Seifert surface to the plane and counted crossings near the projection of their intersection point on the surface, but missed crossings not of this form.

Question 3 (1 attempt): This question tested knowledge of lens spaces, Dehn surgery, and their first homology and homotopy groups. There was one solution, of a good standard. Careful application of the Seifert–van Kampen formula in (a)(iii) and the Mayer–Vietoris exact sequence in (b)(ii) were the more challenging parts of the question.

C5.5: Perturbation Methods

Question 1. Many candidates attempted this question. The early parts of (a) were well answered, but the majority of candidates did not use a formal proof by induction to establish the full asymptotic expansion. In (b) the majority of candidates struggled with establishing the steepest descent contours, though most recognised that the dominant contribution to the integral comes from the region close to the saddle point and could correctly simplify the problem to get the stated result.

Question 2. Many candidates attempted this question. There was a mistake in this question, the term $\partial^2 u/\partial x^2$ should have read $\partial^3 u/\partial x^3$. This did not affect solutions to part (a), and this was very well-answered. The typo affected parts (b)-(d) – all candidates were given marks for making all possible progress.

Question 3. A smaller subset of candidates answered this question. In (a) some marks were lost for not fully justifying where the boundary layer lies. In (b) many candidates struggled to solve the equation for f_0^{outer}, and then solve for the full outer solution. Part (c) was relatively well answered, but very few candidates could properly demonstrate use of an intermediate variable to match the inner and outer solutions in (d).

C5.6: Applied Complex Variables

Question 1

This was by far the least popular question, attempted by only a quarter of candidates, with the lowest average mark. Parts (a) and (b), which were bookwork, were handled relatively well. A few candidates managed part (c), though some did not realise $\bar{\zeta} = 1/\zeta$ when $|\zeta| = 1$ so that $\Re(\zeta) = \Re(1/\zeta)$. No candidate completed part (d), though some managed to get the blowup time, while others managed the volume calculation.

Question 2

This was a popular question, attempted by 88% of candidates. The material was straightforward, but the question a little unfamiliar (certainly for part (a)). Candidates who applied what they had learned, rather than trying to remember calculations, did well. Part (b) was handled better than part (a) in general.

Question 3 This was also a popular question, attempted by 88% of candidates, and had the highest average mark. The material was more challenging than Q2, but the format of the question was familiar, and there were some very good answers. Most mistakes were algebraic. One commom mistake was for candidates to write that the residue of $1/(2k + \mathbf{i})$ at $k = -\mathbf{i}/2$ was 1 rather than 1/2.

C6.1: Numerical Linear Algebra

Question 1 had the fewest attempts. Students who attempted this question generally did well, with the most challenging part being the more elaborate proof of the search direction orthogonality.

Question 2 had unusually high scores due in part to the similarity between parts a) and c) which seemed to cause part c) to be more readily solved than anticipated. Part c) was anticipated to be challenging as Jacobi for eigen-values was only covered briefly in lecture and as it makes use of an unusual variant of Givens rotations; that said students performed remarkably on this part which was impressive.

Qeustion 3 part a) on Householder transforms was standard and solved well by most students with the main omission being not showing the matrices are unitary. Part b) on the singular value decomposition was solved admirably with the main issue being eigenvalues which were zero and completing the matrices with orthogonal columns to span the space. Part c) on the power method was generally well solved, though few students were careful in considering the case of repeated eigenvalues and the eigenvector subspace being greater than one-dimensional.

C7.4: Introduction to Quantum Information

Question 1: This question was the most popular among the students, and those who attempted it performed very well. Parts (a), (b) and (c) posed no significant problems. In part (d), most students demonstrated a solid understanding of quantum circuit analysis, including the phase kick-back mechanism. Some even connected the parity check to the Bernstein-Vazirani problem discussed during lectures. Part (e) did not pose a problem for those who understood the phase kick-back mechanism, but some students struggled to explicitly specify the state of the first register at the output. The final part required interpreting maximally mixed states as an equally weighted statistical mixture of any two orthogonal states, specifically $|+\rangle$ and $|-\rangle$. Most students succeeded in this and correctly identified the probability of the inconclusive answer. However, the second part of (f), in which a can be any binary string of length three, allowed for some interpretation. Some students assumed the second register could be measured, while others assumed it could not. Both assumptions led to different probabilities of the inconclusive outcome, and both answers were accepted if properly justified. Overall, this question was relatively easy.

Question 2: This question was attempted by 23 out of 49 candidates. Overall, the students demonstrated a solid understanding of the core concepts of quantum error correction, including the origin of syndromes and the conditions for detecting and correcting errors. Consequently, nearly all of them provided reasonable answers to all parts of the question. However, due to frequent minor mistakes, only one person achieved full marks. The common mistakes are: missing minus sign for YY, confusing the role of checks and data qubits, not being able to identify the logical X and Z operators and confusing error detection with error correction.

Question 3: Apart from a few small points, most students who attempted this question did very well on the first few parts. Quite a few lost points in (b) for not actually proving that the new Kraus operators did indeed satisfy the orthogonality condition necessary to be Kraus operators. Part (c) was maybe the hardest for most, with few students successfully arguing why the Kraus operators had to be proportional to the identity; some used a clever commutativity argument, and others noticed the idea given in the model solutions. Part (e) was maybe poorly worded, since a lot of students did not justify why sigma' was in general distinct from sigma. As for parts (g) and (h), students tended to either get full marks (often drawing pictures of orthogonal isometries into a larger Hilbert space as non-overlapping subsets) or few marks at all; quite a few just neglected to define the measurements asked for in (g), and some tried to argue for applying V^{-1} instead of $V^{\dagger}$.

C7.5: General Relativity I

The most common combination of questions was Q2 with one of Q1 or Q3, the latter had almost the same number of attempts.

Q1: Part a (i) and (ii) were generally well done being purely book work and things seen in class. There were a few students who struggled with a (ii) but by far the majority got full marks for this part of part a. On the whole a (iii) was poorly done. Only a few students managed to find all three Killing vectors, though most managed to find the obvious one by inspection. Many were confused about how to begin despite being given the equation for a Killing vector in two different forms and some of those that managed to set up the differential equations then struggled to find solutions from these.

b) The vast majority managed to get 7/7 for part b. A common issue amongst those who did not get full marks was not being able to recognise how to combine the derivative on the RHS at the end. This was less a problem with the knowledge of the course and an oversight on the Leibniz rule for derivatives.

c) Only the top students managed to get full marks on this part of the question, though many students picked up a few marks. This looks like a nasty question at first sight but by using part b the student should recognise that b(i) fixes the connection up to the weight w. Some students managed to see this but then did not input the weight w into their answer. If the student managed to get c(i) extending to c(ii) was largely trivial, though a few who did get c(i) did not get c(ii). Some students obtained marks in c(ii) without having done c(i) correctly by observing how to extend c(i) to c(ii).

Q2: This was the most popular question by far and generally the best performance of students came in this question.

a) Generally well done. a(i) was almost always done correctly. A few students struggled to derive the geodesic equations while a few decided to derive the affinely parametrised equations instead despite it being stated to find the non-affinely parametrised one. For a(iii) a few students just stated to make the parameter the proper time without showing anything. Others just stated that if they made $\frac{d\mathcal{L}}{d\lambda} = 0$ it would be affinely paramtrised, this does not answer the question and so they picked up only 1 mark.

b) This was done well by most students. Almost everyone got full marks for b(i). Some lost a mark for b(ii) because they did not justify why they can take the initial conditions that they chose, saying spherical symmetry was sufficient but they instead said 'assume'. The plots were generally correct, though sometimes the interpretation was patchy. The energy needs to be greater than the potential for motion to makes sense, a few students allowed for these types of trajectories.

c) Only a handful of students managed to get full marks. For c(i) none noticed that the light would be reflected from the boundary as it gets there in finite time. Some also solved for the affine parameter rather than the time. c(ii) should have been obvious from their plots in part b and was added to test their understanding. A radial timelike geodesic never reaches $r = \infty$ as can be seen from part b. Despite this, quite a few students managed to find a finite time for the timelike curve to reach $r = \infty$. This included students who had correctly plotted the potentials and correctly identified the motion.

Q3: Quite a few people forgot parts of Birkhoff's theorem and so did not get full marks. For part b) this was seen in the problem sheets. A number of students started trying to solve the geodesic equations despite being told not to assume a geodesic. Other students were on the correct track but then could not form the inequalities correctly and manipulate them. For example they identified correctly that $f(r)$ was negative but then when multiplying by $f(r)$ the inequality remained the same. They would still end up at the correct answer but it was clear that they had fudged the result to match the given answer.

c) Most got 5/5 for this question.

d) Very few managed to perform the coordinate transformation in d(i). Some ended up with a metric that did not make sense since it was degenerate on the horizon which they stated later was not the case. They should have realised this here. (ii) were free marks and most got 2/2. (iii) Not everyone explained this well. They needed to state in the original coordinates one has $U < 0, V > 0$ however in the new metric there is no singularity at $U = 0$ or $V = 0$ and so can be extended to $U, V \in \mathbb{R}$. On the whole this was done well. A few students did not manage to label the diagram correctly and the black hole region and exterior were flipped or in the part where there was no spacetime. These students were the ones who did not score well in on this question and should have used part (ii) to double check their labelling.

C7.6: General Relativity II

Q1 was the most popular, followed by Q2 and then Q3 though there was very little between the numbers.

Q1: Part (a) was done well by most candidates, though a few mixed up the definitions of a(ii) and a(iii).

Many got most marks in part (b). A common mistake in b (i) amongst those who did not get the full marks offered was just to show that a null vector remains null after a conformal transformation making no mention of geodesics. Part b(iv) confused a number of people despite the conserved charge being the same in the Killing vector case, not all of those who gave the conserved charge also said it needed to be a null geodesic.

Part (c) was not done very well. Very few managed to write down the Penrose diagram. There was a worrying number of students who were unable to identify that the radial coordinate was finite in this case. It behaves in much the same way that the θ behaves which should have made them think.

Q2: Parts (a) and (b) were done well overall. Part c was mixed. The strong students managed to do this without issue while the weaker students scored very few marks on this question.

Q3: Part (a) was probably too easy as most who attempted it scored 8/8. There were a few who lost marks because they did not define $h^{\mu\nu}$. Part (b) really separated the stronger candidates from the weaker ones. A number of candidates did not realise that they could use their results from part a to simplify the computations despite the hint to do so.

C7.7: Random Matrix Theory

Question 1 was well done by the students who attempted it. In (a) (iii), few students mentioned the fact that the moments are easily bounded here. Few students appealed to the Wick's theorem which does not apply here, as the distribution is not Gaussian. Part (b) is essentially bookwork, but some students were not precise enough when describing the correspondence with graphs in (i). Parts (i) and (ii) were well done in general. Common mistakes in Part c include the wrong development of the square. The explanation in (ii) of the $1/n^2$ were quite well done. In (iii), perhaps surprisingly, few students computed that the average is 2, using the number of Dyck paths or the Catalan number. The application of Borel-Cantelli was well understood.

Question 2 was attempted by most students. The average was quite high as the amount of bookwork or previously seen material was quite high. In Part a, most students saw the connection with the vandermonde determinant and showed orthogonality of the functions. Most students cited Gaudin's lemma correctly and got the right form for the probability density. Part b was well done, as most spacing got the correct scaling. In Part c, some students did not apply the correlation function to the difference of the angle and that gave them the wrong scaling. Perhaps surprisingly, few students were able to calculate the asymptotic behaviour of the density for small x even though the density was provided in the answer.

Question 3 was attempted by few students (around 10). This is perhaps because it involved Dyson Brownian motion which was covered in lectures but perhaps less than the first two questions. The question was a twist on a problem on the first problem sheet and a good student should have seen the connection. All students that attempted the question got Part (a) as it was simple linear algebra. No student realized that the gap process has a simple form, 2D Bessel process as seen in lecture. Few students applied the Itô's formula correctly. Most students got that the SDE is ill-defined if one starts at 0. Nobody attempted Part (c) even though the density of the OU process was provided. It is easy to write down the integral for the expectation from the density. It would have gotten full mark but nobody attempted it. The PDF is a similar computation. No information on Part (b) was necessary.

Summary: The exam was well done in general. There was a good amount of bookwork and most students did well on this. Some (very few) students struggled with the questions even though they were close to questions done in classes or in lectures.

E. Comments on performance of identifiable individuals

Prizes

Prizes were awarded to the following candidates:

The top prize was awarded to:
Tevz Lotric (St John's College)

One student was highly commended for his dissertation:
Nikolai Maslov (Merton College)

F. Names of members of the Board of Examiners

Examiners:

Prof Christopher Beem (Chair, Mathematical Institute, University Of Oxford)
Prof Alex Schekochihin (Department of Physics, University of Oxford)
Prof John Magorrian (Department of Physics, University of Oxford)
Prof Mark Mezei (Mathematical Institute, University of Oxford)
Prof Martin Evans (School of Physics and Astronomy, University of Edinbrugh)
Prof Toby Wiseman (Blacklett Laboratory, Imperial College London)

Assessors:
Dr Abhishodh Prakash
Dr Adam Caulton
Dr Akshay Yelleshpur Srikant
Prof Alex Schekochihin
Prof Aleks Kissinger
Dr Alessio Lerose
Prof Alexander Lvovsky
Dr Alexandr Duplinskii
Prof Andrei Starinets
Prof Andrew Daley
Prof Andrew Dancer
Dr Andrew Mummery
Prof Andras Juhasz
Prof Andre Henriques
Prof Andre Lukas
Prof Ard Louis
Prof Artur Ekert
Dr Balint Koczor
Dr Benedikt Placke
Prof Bence Kocsis
Prof Caroline Terquem
Dr Christopher Couzens
Dr Chris Hamilton
Dr Chusei Kiumui
Prof Christopher Beem
Prof Cornelia Drutu
Dr David Alonso
Prof David Marshall
Dr David Nadlinger
Dr Davide Spriano
Prof Edward Hardy

Prof Dominic Joyce
Prof Fernando Alday
Dr Gabriel Wong
Dr Georges Obied
Dr Gerben Venken
Prof James Maynard
Dr James Read
Prof James Sparks
Prof Jared Tanner
Dr Jasmine Brewer
Prof Jason Lotay
Prof John Chalker
Prof John Magorrian
Prof John March-Russell
Prof John Wheater
Prof Jon Chapman
Prof Kobi Kremnitzer
Dr Lakshya Bhardwaj
Prof Lionel Mason
Prof Louis-Pierre Arguin
Prof Luis Fernando Alday
Mario Lopez
Prof Mark Mezei
Prof Massimiliano Gubinelli
Dr Matty Hoban
Prof Michael Barnes
Dr Nayara Fonseca
Dr Nick Jones
Dr Nicola Pedreschi
Dr Owen Maroney
Prof Paul Dellar
Dr Peter Drmota
Prof Peter Grindrod
Prof Philip Candelas
Dr Pieter Bomans
Dr Plamen Ivanov
Prof Prateek Agrawal
Prof Renaud Lamboitte
Dr Romain Ruzziconi
Prof Ruth Baker
Prof Sakura Schafer-Nameki
Sam Von Der Dunk
Prof Shivaji Sondhi

Prof Siddharth Parameswaran
Prof Stephen Blundell
Prof Steven Balbus
Prof Steven Simon
Prof Tim Woolings
Prof Xenia de la Ossa
Dr Zhenghao Zhong