Examiners' Report: Final Honour School of Mathematical and Theoretical Physics Part C and MSc in Mathematical and Theoretical Physics Trinity Term 2022

February 28, 2023

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

See Table 1.

	Numbers						
	2022	2021	2020	2019	2018	2017	2016
Distinction	38	42	42	40	25	31	18
Merit	13	10	9	6	n/a	n/a	n/a
Pass	11	12	3	6	17	10	3
Fail	1	3	1	1	0	0	0
Total	63	67	55	53	42	41	21
	Percentages %						
	2022	2021	2020	2019	2018	2017	2016
Distinction	60	63	76	76	60	76	86
Merit	20	15	17	11	n/a	n/a	n/a
Pass	17	18	5	11	40	24	14
Fail	2	4	2	0	0	0	0
Total	100	100	100	100	100	100	100

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**
  No vivas were held.
- **Marking of scripts.**

All dissertations and three mini-project subjects were double-marked. In cases of significant disagreement between marks, the two markers were consulted to agree a reconciled mark.

All written examinations and take-home exams were single-marked according to carefully checked model solutions and a pre-defined marking scheme, which was closely adhered to. A comprehensive independent checking procedure was followed.

B. New examining methods and procedures

Written examinations were all held in person this year in a partially open-book format (except in cases where special circumstances made it necessary for individuals to take invigilated exams online).

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

Notices to candidates were sent on: 18th October 2021 (first notice), 23rd November 2021 (second notice), 25th February 2022 (third notice), 10th March 2022 (fourth notice) and 5th May 2022 (final notice).

The examination conventions for 2021-2022 are on-line at http://mmathphys.physics.ox.ac.uk/students.

Part II

A. General Comments on the Examination

B. Equality and Diversity issues and breakdown of the results by gender

Removed from public version.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 2 and in the Average USM per Formal Assessment graph below. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 2: Statistics for individual papers

Paper	Number of Candidates	Avg USM	StDev USM
Advanced Fluid Dynamics	-	-	-
Advanced Philosophy of Physics	-	-	-
Advanced Quantum Field Theory	32	67	16
Advanced Quantum Theory	25	65	20
Algebraic Geometry	-	-	-
Algebraic Topology	-	-	-
Applied Complex Variables	9	66	11
Collisionless Plasma Physics	-	-	-
Differentiable Manifolds	9	64	4
Dissertation (single unit)	18	72	-
Dissertation (double unit)	25	77	-
Elasticity and Palsticity	-	-	-
Further Functional Analysis	-	-	-
Galactic and Planetary Dynamics	9	67	10
General Relativity I	27	64	18
General Relativity II	8	67	10
Geophysical Fluid Dynamics	8	67	14
Groups and Representations	42	70	15
Homological Algebra	-	-	-
Introduction to Quantum Information	34	74	13
Introduction to Schemes	-	-	-
Kinetic Theory	7	70	11
Networks	12	70	7
Numerical Linear Algebra	-	-	-
Perturbation Methods	16	66	12
Quantum Field Theory	58	70	12
Quantum Matter	15	69	22
Radiative Processes and High Eng. Astro.	-	-	-
Random Matrix Theory	-	-	-
Riemannian Geometry	-	-	-
Solid Mechanics	-	-	-
Stochastic Differential Equations	-	-	-
String Theory I	24	70	9
Supersymmetry and Supergravity	7	80	14
Topics in Fluid Mechanics	-	-	-

The number of candidates taking each homework-completion course is shown in Table 3. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 3: Numbers taking each homework completion course

Paper	Number of Candidates	Percentage completing course
Advanced Fluid Dynamics	-	-
Astroparticle Physics	6	100
Collisionless Plasma Physics	-	-
Collisional Plasma Physics	-	-
Conformal Field Theory	25	100
Cosmology	13	100
Galactic and Planetary Dynamics	-	-
Group and Representations	41	98
High Energy Density Physics	-	-
Kinetic Theory	9	100
Nonequilibrium Statistical Physics	7	100
Quantum Field Theory in Curved Space Time	-	-
Quantum Matter I	7	100
Quantum Matter II	7	100
Quantum Processes in Hot Plasma	-	-
Renormalisation Group	10	100
Soft Matter Physics	-	-
String Theory II	10	100
Supersymmetry and Supergravity	11	100
Symbolic, Numerical and Graphical Scientific Programming	12	100
The Standard Model and Beyond I	8	100
The Standard Model and Beyond II	8	88
Topological Quantum Theory	20	100

D. Assessors' comments on sections and on individual questions

Advanced Fluid Dynamics

Question 2:

There was a mistake in part (d) of the question, $\mu \mathbf{e} \cdot \mathbf{n}$ should have been $5\mu \mathbf{e} \cdot \mathbf{n}$ in the first displayed equation for part (d).

Noone completed part (a) as intended, using the divergence theorem to convert the volume integral into a surface integral, substituting $\mathbf{u} = \mathbf{e} \cdot \mathbf{x}$ on the surface, then using the divergence theorem again to equate the surface integral of $n_i x_j$ with the volume integral of $\partial_i x_j = \delta_{ij}$.

Noone completed part (b) as intended, using the divergence theorem as in part (a) with $\mathbf{u} = \mathbf{e} \cdot \mathbf{x}$ on the outer boundary of the fluid, but now with $\mathbf{u} = \dot{\mathbf{D}} \cdot \mathbf{x}$ on the boundary of the particle. Using the divergence theorem again for the volumes enclosed by the two surfaces, with consistent signs for the normals, gives the result. The integral over the outer boundary becomes an integral over the whole volumes enclosed by this surface, giving $|V|\mathbf{e}$ instead of $|V_\mathrm{f}|\mathbf{e}$ even though the volume occupied by fluid is $|V_\mathrm{f}|$.

Most candidates tried to superimpose flows using part (a). However, the presence of the particle changes the flow in the fluid outside the particle, so one cannot take $\mathbf{u} = \mathbf{e} \cdot \mathbf{x}$ in the fluid.

Part (c) was done correctly by everyone.

Writing $\mathbf{u} = \dot{\mathbf{D}} \cdot \mathbf{x} + \tilde{\mathbf{u}}$ as instructed, the first part of the solution ($\mathbf{u} = \dot{\mathbf{D}} \cdot \mathbf{x}$) satisfies the Stokes equations and has strain rate $\dot{\mathbf{D}}$. The flow $\tilde{\mathbf{u}}$ thus satisfies the Stokes flow problem in the hint with $\tilde{\mathbf{u}} = 0$ on $|\mathbf{x}| = a$, and $\tilde{\mathbf{u}} \sim (\mathbf{e} - \dot{\mathbf{D}}) \cdot \mathbf{x}$ as $|\mathbf{x}| \to \infty$. The outer boundary can be taken to infinity as the particle is small. The stress due to this flow satisfies $\tilde{\boldsymbol{\sigma}} \cdot \mathbf{n} = 5\mu(\mathbf{e} - \dot{\mathbf{D}}) \cdot \mathbf{n}$ on $|\mathbf{x}| = a$. We can apply boundary conditions at $|\mathbf{x}| = a$ because the deformations of the sphere are negligibly small. The total stress in the fluid just outside the particle boundary is then

$$\boldsymbol{\sigma} \cdot \mathbf{n} = (-p\mathbf{I} + 2\mu\dot{\mathbf{D}}) \cdot \mathbf{n} + \tilde{\boldsymbol{\sigma}} \cdot \mathbf{n} = -p\mathbf{n} + 5\mu\mathbf{e} \cdot \mathbf{n} - 3\mu\dot{\mathbf{D}} \cdot \mathbf{n}.$$

Equating this with $\boldsymbol{\sigma} \cdot \mathbf{n} = (-p\mathbf{I} + 2G\mathbf{D}) \cdot \mathbf{n}$ just inside the particle boundary, for all directions of the normal $\mathbf{n}$, gives the second displayed equation.

Part (e) received one complete attempt and some partial attempts. The tensors $\mathbf{e}$, $\mathbf{D}$, $\dot{\mathbf{D}}$ are all traceless, so $\mathrm{Tr}\,\boldsymbol{\sigma} = -3p$ in both the fluid and the particle. Taking the traceless part of $\boldsymbol{\sigma}$ just removes the pressure,

$$\mathbf{S} = 2\mu\mathbf{e} + \phi(2G\mathbf{D} - 2\mu\dot{\mathbf{D}}) = 2\mu(1 - 5\phi/3)\mathbf{e} + (10/3)\phi G\mathbf{D},$$

using the second result in part (d) to eliminate $\dot{\mathbf{D}}$. Taking the time derivative and eliminating $\dot{\mathbf{D}}$ again gives the required result. No further approximations are needed. As $\tau \to 0$ we recover the Einstein viscosity for a suspension of rigid particles.

Advanced Quantum Field Theory

Question 1: related to a 1–loop calculation of the production of a Higgs boson via $\gamma\gamma$ collisions. This was completely unseen. Parts (i) and (ii) were relatively straightforward questions in order to set the problem up, and were answered well. Part (iii) required that the trace in the numerator be evaluated, using the provided γ matrix identities and the student's knowledge about the properties of transverse photon polarizations. A good number of students achieved full marks or close to this, while some knew the procedure to follow but made some algebra mistakes (certainly possible given the trace manipulations required). A small number failed to answer the question or did not know how to handle the trace at all. Parts (iv) and (v) followed on from part (iii) but were asked in such a way that they could be fully answered on the basis of the quoted result alone. Part (iv) was answered well in general, with many students noting what was needed in order for the integral to be finite. However, almost none could explain why this should be the case. As with part (iii) some students either did not answer or recognise what was needed. Part (v) was clearly more challenging, with no students achieving full marks or close to it. While the requirement that $B = C$ was observed by many students, the full reason for this was not.

Question 2: (a) was bookwork relating to the running coupling in QFT. It was answered completely by almost all students. Question 2 (b) related to various tree–level QCD and QED scattering processes, requiring that the scattering amplitude be evaluated, as well as colour factors be evaluated. Part (i) involved a relatively standard amplitude calculation. Many students achieved full marks or close to it. Some students struggled to get the colour factors correct, or had time issues potentially. A small minority could not identify the contributing diagram and hence set the question up at all. Part (ii) simply required that the relevant diagrams be identified and the corresponding contribution in the amplitude explained. It was answered well by almost all students. Part (iii) required that the contributing diagrams be identified and the corresponding colour factors evaluated. Many students achieved full or good marks on this. A fair number did not get the full result out, often not getting the colour factor for the interference correct. A small number could not identify that the colour factors are different between the terms.

Question 3: (a) related to spontaneous symmetry breaking. Part (i) was bookwork and answered well by almost all students. Part (ii) related to the breaking of a SO(3) symmetry, with the general $SO(n)$ case being familiar from the lectures. Many students achieved full marks. Part (iii) was unseen, but followed a similar methodology to (ii). The standard of answer was more mixed here, with some students achieving good marks, but a good number struggling to deal with the new term in the potential or set the problem up correctly. Question 3 (b) was bookwork, and was answered well by the majority of students.

Advanced Quantum Theory

Q1 A question studying the sawtooth Ising chain using the transfer-matrix method. For part (a), full marks were given for any reasonable derivation, although students were penalised 1 mark for not giving some reason why the transfer matrix did not involve the apex spins $\bar{s}_i$ as was explicitly requested in the problem. For part (b), students should have been able to start with the transfer matrix specified in the problem statement of part (a) to compute the free energy. Several students lost 1 mark for not recalling that there are $2L$ spins in the system (even though this too was stated in the question), which leads to a free energy twice as large as the correct answer. (However, students were not subsequently penalised for incorrect answers that resulted solely from propagating this error downstream.) On part (c), many students missed the simple expression $s = -\partial f/\partial T|_L$ for the entropy density, and many also got bogged down by algebraic complexities. A useful step is to first simplify the piece of the free energy density inside the logarithm (i.e., the partition function) in the low- and high-temperature limits for the two cases $\bar{J} = J$ and $\bar{J} > J$ *before* taking the temperature derivative, which vastly reduces the extent of mathematical manipulations required. While a rather disconcertingly diverse range of answers were given for the low-temperature entropy density (zero, finite, and divergent in both correct and incorrect permutations thereof), nearly all students realised that the high-temperature entropy density is universal and is just that of a free Ising spin. Finally, for part (d) while most students correctly identified the set of ground states for both the two cases, far fewer students correctly explained how these translate into a finite $T \to 0$ entropy density when $\bar{J} = J$ but not when $\bar{J} > J$, required for full marks.

Q2 A question applying the Holstein-Primakoff technique to an easy-plane ferromagnet with a Dzyaloshinskii-Moriya type term, with a final piece exploring a situation where these may not have the same symmetry axes. On part (a), students were penalised for not clearly explaining why a constraint was needed in the Holstein-Primakoff approach. Furthermore, given that the constraint was specified in the problem, for full marks two additional complications should have been identified. In part (b), the reason $u \gg 1$ was given only to identify the preferred quantization axis for the HP approach. Some students chose to use this fact to drop terms in H. Although this was unnecessary to completing the problem, full marks were given except when students completely dropped the $J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)$ term, which is clearly incorrect (in contrast to, say, writing $J(1+u) \approx uJ$ for $u \gg$). Wherever possible without altering the meaning of the problem, students were not penalised for this choice downstream. Modulo these incorrect assumptions most students successfully worked out the basic forms of the two coefficients A, B, but often with sign errors or omitted prefactors of J and/or S which were required for full marks. Most students successfully implemented the Fourier transform piece of part (c), although the answers to the conceptual questions of the ground/low-lying excited states were of widely varying quality and level of rigor. On part (d), only a handful of students realized the need to first determine the minimum of the dispersion $\epsilon(k)$ which occurs at some $k^* \neq 0$, in order to successfully obtain the critical value of u for the gap-closing transition. Finally, in part (e), many students attempted to give qualitative reasoning rather than an explicit calculation as requested in the problem and suggested in the hint. Such cases were nevertheless not penalised as long as a serious attempt was made to give a complete explanation.

Collisionless Plasma Physics

The questions on plasma waves (Q1 and Q2) were handled really well by the candidates, all of whom showed excellent grasp of the material and scored nearly full marks.

The KMHD question (Q3) proved far more difficult. In part (a), candidates had a basic grasp of what needed doing, but the long calculation necessary to do it defeated, to varying degrees, all. In part (b) again, there were few conceptual difficulties (although the interpretation of the three temperature evolution terms as due to compressional heating, heat flux, and pressure-anisotropy, or "viscous", heating might have been articulated clearer), but considerable difficulties with algebra. There was a minor error in the exam script — "left-hand side" should have been "right-hand side" — but all candidates figured that out. Answers, or lack of answers, to part (c) suggested that only one candidate really grasped how Braginskii viscosity arose from the collisional limit of CGL equations.

Groups and Representations

Question 1: This question was attempted by 38 students. Parts (a), (b) and (c) were, on the whole, carried out well, with occasional calculation errors, particularly for the characters, affecting performance. Part (d) and the requested interpretation in part (e) caused more problems but were still completed well by a good number of students.

Question 2: A very unpopular question attempted by only 4 students. Much was done well but in some cases there were problems correctly identifying the positive simple roots.

Question 3: This question was attempted by all 42 students. Most parts were done very well. The main problems were causes by incorrect applications of Schur's Lemma in part (b). Many also struggled to come up with sensible physical interpretations in parts (d) and (e).

Question 4: This question was attempted by all 42 students. The routine calculations in parts (a), (b) and (c) were done very well. A considerable number of students failed to write down the correct projection matrix in part (d). The application to unification in part (e) also caused problems and only few students were able to identify the correct mulitplets.

Kinetic Theory

Question 1: Most candidates made little progress with parts (c) and (d). There was a mistake, a missing $\cdot\mathbf{n}$ in the displayed equation for part (d). Though regrettable, this seemed to cause little difficulty.

For parts (b) onwards it was essential to distinguish between the half-spaces $\mathbf{v}\cdot\mathbf{n} > 0$ and $\mathbf{v}\cdot\mathbf{n} < 0$. Common mistakes were to omit integration ranges from integrals, and to assume that the reflection formula ($\star$) held for all velocities $\mathbf{v}$, not just for reflected particles with $\mathbf{v}\cdot\mathbf{n} > 0$.

(a) Almost all candidates completed this part successfully, though few took the most direct route of multiplying the Boltzmann equation by $1 + \log f$. A few candidates assumed the BGK collision operator, and so did not establish the result for the Boltzmann collision operator as required. The best solutions noted that 1 is a collision invariant, and that $B(V, \theta) \geq 0$.

(b) The most complete answer observes that, for each velocity $\mathbf{v}'$ directed towards the boundary, $R(\mathbf{v}', \mathbf{v})$ is a correctly normalised probability distribution for the reflected velocity $\mathbf{v}$ directed away from the boundary. Several candidates tried to use the given formula ($\star$) for particles propagating both towards and away from the boundary. Some omitted the $\mathbf{v}\cdot\mathbf{n}$ from the mass flux $\int \mathbf{v}\cdot\mathbf{n}\, f \,\mathrm{d}\mathbf{v}$.

(c) This was found much more difficult than expected. To match the hint, define $g(\mathbf{v}) = f(\mathbf{v})/f^{(0)}(\mathbf{v})$ and consider, for $\mathbf{v}\cdot\mathbf{n} > 0$,

$$C\left(\frac{f(\mathbf{v})}{f^{(0)}(\mathbf{v})}\right) = C\left(\frac{1}{|\mathbf{v}\cdot\mathbf{n}| f^{(0)}(\mathbf{v})} \int_{\mathbf{v}'\cdot\mathbf{n}<0} R(\mathbf{v}', \mathbf{v})|\mathbf{v}'\cdot\mathbf{n}| f(\mathbf{v}')\,\mathrm{d}\mathbf{v}'\right).$$

Many candidates tried to treat $f(\mathbf{v}')/f^{(0)}(\mathbf{v})$ as $g(\mathbf{v}')$, but $f^{(0)}$ is then evaluated at $\mathbf{v}$ not $\mathbf{v}'$. Instead, we must multiply and divide by $f^{(0)}(\mathbf{v}')$, giving

$$C\left(\frac{f(\mathbf{v})}{f^{(0)}(\mathbf{v})}\right) = C\Bigg(\int_{\mathbf{v}'\cdot\mathbf{n}<0} \underbrace{R(\mathbf{v}',\mathbf{v})\frac{|\mathbf{v}'\cdot\mathbf{n}|f^{(0)}(\mathbf{v}')}{|\mathbf{v}\cdot\mathbf{n}|f^{(0)}(\mathbf{v})}}_{w(\mathbf{v}')}\ \frac{f(\mathbf{v}')}{f^{(0)}(\mathbf{v}')}\,\mathrm{d}\mathbf{v}'\Bigg).$$

This defines a correctly normalises weight function $w(\mathbf{v}')$ over the half-space $\mathbf{v}'\cdot\mathbf{n} < 0$, so the hint gives

$$C\left(\frac{f(\mathbf{v})}{f^{(0)}(\mathbf{v})}\right) \leq \int_{\mathbf{v}'\cdot\mathbf{n}<0} R(\mathbf{v}',\mathbf{v})\frac{|\mathbf{v}'\cdot\mathbf{n}|f^{(0)}(\mathbf{v}')}{|\mathbf{v}\cdot\mathbf{n}|f^{(0)}(\mathbf{v})}\,C\left(\frac{f(\mathbf{v}')}{f^{(0)}(\mathbf{v}')}\right)\mathrm{d}\mathbf{v}'.$$

We can now multiply by $|\mathbf{v}\cdot\mathbf{n}|f^{(0)}(\mathbf{v})$, which is independent of $\mathbf{v}'$, and integrate both sides over $\mathbf{v}\cdot\mathbf{n} > 0$. Evaluating the $\mathbf{v}$ integral on the right-hand side using property (ii) of R gives the required result.

(d) Few candidates spotted that the function $C(g) = g\log g$ gives $\mathbf{J}$ from part (a). Many used $C(g) = -\log g$, which gives $f^{(0)}\log f$ instead of $f\log f$.

The correct Maxwell–Boltzmann distribution to use is $f^{(0)}(\mathbf{v}) \propto \exp(-|\mathbf{v}|^2/2\,\theta)$ for the stationary boundary, not $f^{(0)}(\mathbf{v}) \propto \exp(-|\mathbf{v}-\mathbf{u}|^2/2\,\theta)$ centred on the fluid velocity $\mathbf{u}$ adjacent to the boundary. We know that $\mathbf{u}\cdot\mathbf{n} = 0$ from part (b), but the tangential fluid velocity generally does not vanish.

Question 2: Performance on Question 2 was variable. The question was probably on the harder side but some students did acquit themselves extremely well—proof of the principle that this was doable.

Part (a) was perfectly standard and was done perfectly by all.

Part (b) required realising that, since $\omega_{\mathrm{p}i} \ll \omega_{\mathrm{p}e}$, the term containing the instability could only be non-negligible if $|ku_i - ip| \ll p$. An expansion $ip = ku_i + i\delta p$ was then to do the trick.

Part (c) had an easy bit and a harder bit. The evolution of the electron energy was easy and standard (just assume $kv, \gamma_k \ll \omega_k \sim \omega_{\mathrm{p}e}$), and quite a few figured it out. The evolution of the ion energy required integration by parts and some algebra substituting for ω_k and γ_k from the solution obtained in Part (b). Very few could do this efficiently or arrived at the right answer, but several did get the basic idea right.

Part (d) was an "essay question": no calculations needed, just a realisation that the beam was the energy source from which the instability transferred energy into Lamgmuir waves, whose energy was $2\mathcal{E}$ (electric + kinetic). The kinetic part of that energy showed up as an increase in the kinetic energy of the electron distribution.

Part (c) required qualitative thinking—often hard. As the system evolves, the ion beam slows down while the electron distribution gets wider, so the "gap" between them becomes less pronounced. It is reasonable to guess that the instability saturates when that gap roughly "fills up", i.e., when $v_{\mathrm{th}e} \sim u_i$. So the saturation level is something like $\mathcal{E} \sim m_e n_e v_{\mathrm{th}e}^2/2 \sim m_e n_e u_i^2/2$.

Question 3: (a) Some candidates did not recover the correct definition of the instantaneous Hamiltonian, $H_{\mathrm{d}}(\mathbf{x}, \mathbf{v}) = |\mathbf{v}|^2/2 + \int \mathrm{d}\mathbf{x}'\mathrm{d}\mathbf{v}'\, U(\mathbf{x}, \mathbf{x}')F_{\mathrm{d}}(\mathbf{x}', \mathbf{v}')$.

(b) This question did not cause any difficulty.

(c) Many candidates forgot to mention that $\partial F_0/\partial t = -\langle[\delta F, \delta\Phi]\rangle$, i.e. this term is second-order in the perturbations, so that it can be neglected in the system's first-order evolution equation.

(d) The manipulations of the equations were done correctly, except, in some cases, for the explicit mention that $\mathrm{Im}(\omega) > 0$ large enough ensures the appropriate vanishing of $\mathrm{e}^{\mathrm{i}\omega t}\delta F(t)$ for $t \to +\infty$.

(e) Some candidates missed the fact that $F_0 = F_0(\mathbf{J}, t)$ allows one to average $\partial F_0/\partial t$ with respect to θ.

(f) Some candidates missed the mass prefactor, m, in the thermal equilibrium, $F_0(\mathbf{J}) \propto \mathrm{e}^{-\beta m H_0(\mathbf{J})}$.

Quantum Field Theory

Question 1: This question was mostly very well done. Some candidates failed to take note of the identities given at the end of the question and embarked on computing loop integrals from scratch. Candidates were oftenunsure about when external momenta could be ignored in computing the counterterms; only in the case of δ_z need the external momentum be retained.

Question 2: This question was mostly very well done, but there was a wider variety of errors than in the first question. Many candidates were unable to get the Feynman rules completely correct; the most common errors were failing to notice that the mass of the auxiliary field is 1, and getting the wrong sign for the vertex rule. Some were confused about when antisymmetry is required (in states with identical fermions, but not fermion/anti-fermion states). Most candidates who managed the last part did it by graphical analysis rather than using the path integral which is easier.

Question 3: This was the question candidates found hardest, and some were clearly running short of time. A number of candidates obtained the correct result in part a) only to then erroneously conclude that it is zero by being careless with Dirac delta functions. Many candidates struggled with part b) because they did not realise that the result obtained in part a) should be used.

Quantum Matter

Question 1:

Part (a) was done fairly well by most people. A question like this occurs almost every year and I think by this time students expect it. Some marks were lost in not answering all parts of the question. A fair number of students did not manage to handle the integration by parts. Making mistakes in the integration by parts did not cost many marks here, but often confused students on later parts of the question where many marks were lost.

A large fraction of those that got (a) correct did manage to get (b) as well. However many did not manage the simple manipulation to put it in the form of (c).

Part (d) confused a lot of students. Just because expectations come out equal does not mean the operators are equivalent. A few students got full marks on this, a perfect answer required realizing that "off-diagonal" terms would not be modeled by the proposed approximation.

(e) On this part many students got some marks by discussing the Landau criterion and guessing that the disperion is quadratic. Very few got full marks by *correctly* arguing that the quasiparticle excitations would be sub-linear. Invoking an incorrect conclusion from part d was only give partial credit.

Question 2:

Parts (a) and (b) of this problem are exact duplicate of a homework assignment so all students should have gotten it right. Unfortunately, a fair number did not. Part (c) differs from the homework only in that we need to use the 2D fourier transform of the coulomb interaction rather than the 3D fourier transform. (The actual fourier transform is given to the students, but some did not use it).

There were a good number of students who essentially got everything correct up to the last part of part (d). Students realized that a diverging velocity is problematic, but very few students realized that this stems from the long range nature of the Coulomb interaction. No students realized that the fundamental mistake is that we are using an ″instantaneous″ approximation for the coulomb interaction whereas the electromagnetic waves that carry the coulomb interaction have finite velocity that we usually ignore.

Radiative Processes and High Energy Astrophysics

Question 1: overall students were able to solve this question reasonably well. Nevertheless I found 3 recurrent issues: when determining the number density in part d, students assumed filaments were aligned with the line of sight, which is the least probable configuration. Students did not fully justify why stimulated emission could be ignored in part c. Although the final formula giving the number density as a function of equivalent width was correct, students almost always got the order of magnitude of the final number wrong. This was mostly to errors in unit conversion.

Question 2: The average mark for this question was very similar to Q1. I do not see any particular patterns in the responses. The students did not seem to grasp the diffuse shock acceleration can explain the highest particle energies ONLY if DE/E u/c. Otherwise, the answers show a reasonably good understanding of this topic.

Supersymmetry and Supergravity

Question 1: The overall students' performance on this question was good. Parts (a), (b), (c) cover standard material, and almost all students gave correct answers. Part (d) is a problem that the students had not seen during the lectures or in the homework assignments. All students were able to follow the logic of the question, but a few made algebraic mistakes in manipulating Grassman elds/variables.

Question 2: The students' performance was good for the most part. Parts (a) and (b) were standard, and received good answers overall. In part (c), a couple of students were not able to draw the conclusion that no SUSY vacua are found in the second model, despite writing down the correct F-term equations. Part (d) was about the eld content of a vector multiplet of 4d $\mathcal{N} = 1$ supersymmetry. Only a couple of students were confused about representations of the component elds, with all others giving the correct answer. Part (e) involves a simple proof in superspace, and all but one student indicated the correct steps involved in the proof. Part (f) proved to be more challenging for the students. Overall, the students were somewhat confused by the question on the dimension of moduli space; most students correctly counted degrees of freedom, and subtracted gauge redundancies and the D-term equation, but some applied gauge invariance twice, or failed to combine these ingredients correctly. Not all students attempted the last bullet point in the question (which was a new problem that was not covered in class/homework).

C2.2: Homological Algebra

Question 1: Question 1 was mostly done well by the students. The task of showing that left derived functors are additive functors was somewhat ambiguous: some students interpreted it as asking to show that it's a functor, while some others interpreted it as asking to show that it's additive.

Question 2: Many candidates forgot to check in (a)(iii) the categorical equivalence on morphisms (not just on objects). The most di cult part was (c), but also the implication 3 to 1 in (b) caused di culties.

Question 3: Part (a) and (b) were mostly done well, but parts (c) and (d) turned out to be rather dichotomic: 3 students solved them very well, while the others did essentially nothing.

C2.6: Introduction to Schemes

All candidates did exercise 1, and then the candidates were roughly split 50-50 in choosing exercise 2 or 3. The average raw marks for exercises 2 and 3 were roughly equal, both being 3 raw marks less than the average for exercise 1.

Question 1: (b) many candidates only proved the property at the level of topological spaces, without considering sheaves (in particular, without using the assumption that an open subscheme structure was chosen). Many students did not remember the bookwork for (d)(i), essentially all candidates got the first counterexample for (d)(ii) but only a few managed to find the second.

Question 2: (a) some confusion in writing down the complex correctly, or losing one raw mark for not saying why it suffices to just consider a cover by two basic open sets; (b) was mostly fine; (c) not many candidates explained how the maps in the short exact sequence were defined; (d)(iii) only very few candidates considered the O(k) bundles for suitable k.

Question 3: (b)(i) most candidates were not careful about the issue that epimorphisms are not necessarily surjective on sections, here the key is to consider a generator of L(U) as a free O(U)- module on a small enough neighbourhood U; (c) was mostly fine; (d)(i) again identifying L(U) with O(U) by a choice of generator makes this part easier; (d)(iv) not many candidates checked that the two open subsets cover X, and that the two maps agree on the overlap; (d)(v) not all candidates noticed that the two required functors were already constructed in (c) and (d)(iv), so it was sufficient to say that one checks the constructions are natural and inverse to each other.

C3.1: Algebraic Topology

Essentially everyone chose Exercise 1, and there was a 50-50 split in choosing ex.2 or 3.

Question 1: (a) candidates often just wrote down the answer for the cup product for T^2, without explaining how they used (if they did) the Künneth theorem to compute it; (b) many candidates did not spot that it was enough to apply the projection to the homotopy, because they never wrote down the actual homotopy map; (c) after showing injectivity of the pull-back of the projection map; candidates sometimes did not explain why the SES splits; for the cup product part the key was to consider the unit; for the last part not many candidates realised that the non-commutative cup product from part (1)(a) (using projection to one circle factor) provided a counterexample.

Question 2: (a) generally fine; (b) some minor slips e.g. not noticing that G/2G=0 or Hom(Z/2,G)=0; (c) all candidates wrote a correct fundamental cycle, but very few finished the exercise: some candidates did not draw the barycentric subdivision correctly, some candidates guessed a chain that works but miscalculated the cap product (which needs to be separately calculated, by linearity, for each of the two faces).

Question 3: (a) several candidates wrote down Poincaré duality as a cap product involving homology and cohomology, instead of the requested bilinear form on cohomology (working modulo torsion, in complementary dimensions, and using cup product); acceptable was also to use cup product using coefficients in a field, although the answer to the second half of the question was then harder or incomplete); (b) some candidates forgot to reduce the homology of the quotient when calculating the relative homology; (c) many good answers here, the last part can be done in several ways (either by course methods considering the pull-back on cohomology using that H^2 of the torus is generated by H^1 classes which pull-back to zero in $H^2(S^2)$, or using homotopy methods: the second homotopy group of a torus is zero or more explicitly: lifting a map from S^2 to the torus to the universal cover R^2 of T^2 and then using that R^2 is contractible).

C3.3: Differentiable Manifolds

Question 1: This was a popular choice of question. Part (a) was bookwork and done well. Part (b) was usually done well by students, with marks usually only being lost due to lack of justifcation as to why the map is a submersion. Part (c) was bookwork and typically done well. There was a mixed response to part (d). In (d)(i), the most common reason for losing marks was not justifying suffciently why the maps are embeddings (and not just immersions) and omitting the argument as to why the curves are disjoint. In (d)(ii), the usual approach was to draw a diagram, which yielded the correct minimal distance, but often students failed to correctly identify the points where the minimum is attained. In (d)(iii), the main issue was in showing the linking number is positive, which few students succeeded in justifying.

Question 2: This was a popular choice of question. Part (a) was bookwork and usually done well. Part (b) was bookwork or seen material and usually done well. Part (c) was challenging for students. Some recognized they should use the definition of the Lie derivative and the earlier parts of the question, but they did not reach the desired conclusion. Part (d) had a mixed response from students. Part (d)(i) was either done very well or was found to be difficult. The main issues were that students were not able to spot the solutions to the ODEs defining the flow, and that they missed out the part about the curves preserved by the flow. Part (d)(ii) was understood conceptually, but often led to computation errors.

Question 3: This was the least popular question. Part (a)(i) was bookwork and done well. Part (a)(ii) produced a mixed response. Students who attempted it usually got the key idea, but marks were lost in justifying the argument. For part (a)(iii), the "if" part of the statement was done well, but the "only if" part proved challenging. Students who attempted it followed the hint for that part, but were not able to reach the desired conclusion. Most students missed out the final part of (a)(iii) entirely. Part (b) was bookwork and usually done well, with marks only typically lost for not justifying why the pullback induces linear maps on cohomology. Part (c) was again bookwork and done well. Part (d)(i) was either done well with students only losing marks in justification or it was challenging, which was the case for the majority of students. Students who attempted (d)(ii) did it correctly, only losing marks in showing that the classes are linearly independent.

C3.4: Algebraic Geometry

Question 1: All students answered this question,. There were no issues with the bookwork in (a) and (b). For (c), the standard examples included unions of hypersurfaces or of disjoint points, but a full solution needed the comment that as the base field is algebraically closed, it has infinite cardinality, so an arbitrarily large finite set of disjoint points or hypersurfaces can be found. (d) was largely done well; some candidates failed to realise that the quadric is also isomorphic to the affine line as an abstract variety. (e) caused more issues; the plane component was found by many candidates but the identification of the other component was less straightforward.

Question 2: Most candidates answered this question. In (a), one issue that lead to the loss of a mark was if students failed to explain that projective morphisms are defined locally, or if they failed to mention that the homogeneous polynomials defining them should not have common zeros. (b) was done well. In (c), many candidates found the right open sets, but sometimes failed to argue that they are affine or the proof that the relevant morphisms give an isomorphism had gaps. (d) and (e)(i) were generally done well. For (e)(ii), many candidates thought the answer was yes, though the strongest answers gave a full argument for the fact that S is singular so cannot be isomorphic to the projective plane.

Question 3: Only 4 candidates answered this question. From the scripts it was clear that several others attempted this question, but got stuck in (b)(i) so moved to the other questions. Of the 4 students who carried on, 3 gave substantially complete answers, whereas one did not get very far.

Overall, the average mark on this paper was lower than in previous years, indeed the questions were a little harder than in some previous years, leading to better differentiation of candidates.

C3.11: Riemannian Geometry

Question 1. Part (a) was bookwork and typically done well. Though part (b) was seen and usually done well, marks were sometimes lost because students did not show that the second fundamental form is symmetric. Part (c) was either done almost perfectly or else students found it very challenging. The common issue was to use the minimality condition correctly and compute the appropriate second fundamental form terms arising from the Gauss equation.

Question 2. Part (a) was either done well or the common issue was to try to use the first variation formula and assume the curve was a geodesic, rather than use the hint and the minimizing property. Part (b)(i) was essentially bookwork and typically done well. Students who attempted (b)(ii) invariably had little or no problems with it.

Question 3. Part (a) was bookwork and usually fine. Most students understood well what to do in (b). The only common issues in (b) were not explaining why the exponential map is surjective (using Hopf–Rinow) and why the metric defined in (ii) is complete (again, using Hopf–Rinow). Part (c) proved challenging, with most students not realizing that they had to look at Jacobi fields on the round sphere and relate them to Jacobi fields on the product.

C3.12: Low-Dimensional Topology and Knot Theory

Solutions for Question 1 were generally good, with some candidates failing to require cobordisms to be compact in 1(a)(i). Solutions for 1(a)(ii) were essentially all correct. In 1(a)(iii), several candidates failed to check inverses. In (b)(i), candidates usually had the right idea. Part (b)(ii) proved to be more difficult, but there were several different correct approaches among the solutions, including doubling and the long exact sequence of a pair. Solutions for (b)(iii) were typically correct. There were essentially no complete solutions for part (c), but many partial results. Showing that the connected sum of an even number of copies of the projective plane is null-cobordant was usually missing.

Overall, there were lots of good solutions for Question 2. In part (a), some people forgot to require a Seifert surface to be compact. In part (b), many candidates got the wrong Seifert matrix, due to miscalculating linking numbers. This did not affect the marks given for (b)(iii) and (c). Most solutions for (c) were correct.

There was just one solution for Question 3, which was essentially correct.

C4.1: Further Functional Analysis

Question 1 This question was attempted by all candidates. (a) was often well done, though often with rather convoluted answers in (ii), and lengthy arguments in (iii). Many candidates missed that given $x+Y \in \mathrm{B}^0_{x/y}$ in (ii), the definition of the norm gives $y \in Y$ with $\|x+y\| < 1$ which has $T(x+y) = x+Y$. A number of candidates gave good answers to (b)(i), either using Hahn-

Banach to obtain norming functionals to directly show that $\{x : \|x\| > a\}$ is weakly open, and others quoting the result from the course that norm closed convex sets are weakly closed to see that $\{x : \|x\| \leq 1\}$ is weakly closed.

Part (b)(ii) caused difficulties. Few candidates used the result from the course that 0 is in the closure of S_X if X is in finite dimensional, which quickly shows that if the norm is weakly continuous, then the space must be finite dimensional.

(c)(i) proved challenging for many, but a number of strong and creative answers where produced showing excellent functional analytic skills. It was intended that candidates might take $x \in B_x$ and then consider $B_X(2) + (x + Y)$ which is a weakly closed (as it is norm closed and convex) subset of $B_X(2)$ which is weakly compact as X is reflexive. Then taking an element which attains the in mum of the norm on this set does the job. One very nice alternative answer noted that as

$$T(B_X) \subset \overline{T(B_X)} \subset \overline{T(\overline{\mathrm{B}_X^0})} = \overline{T(\mathrm{B}_{x/y}^0}$$

(using continuity at the first equality) it suffices to show that $T(B_X)$ is norm closed. They then did this as B_X is weakly compact (by reflexivity) and T is weakly continuous. While a number of candidates noted that a non-reflexive space, such as ℓ^1 , would be needed in (c)(ii), few turned this into a counter example. (d)(ii) saw few attempts, probably as candidates where short of time.

Question 2 (a)(i) and (ii) where well done (though often with slightly longer than expected answers for (ii) - few candidates noted that if $p_C(x) \geq 1$, then $x \in C$ as C is closed). In (iii) candidates could have saved time by simultaneously showing the required equivalence along side demonstrating that p_C is a norm (and many candidates didn't use the conditions in the course for when p_C is a norm). (iv) caused difficulties to many candidates, with only few making much progress with the Hahn-Banach separation argument (or the converse, where one should use the failure of the condition to give an explicit weak*-open neighbourhood of some f with $|||f|||_{X*} > 1$).

Part (b)(i) was generally well done, and many candidates showed that $|||f|||_{X*}$ is not strictly convex. Few noticed that it was also necessary to check that the ball $\{f \in X^* : |||f||| \leq 1\}$ is weak*-closed in the topology coming from the original norm on X^* so that (a)(iv) could be applied to see that this is the dual norm of the associated norm p_C on X.

Question 3 This question was not popular. Part (a) was typically well done, as was part (b)(i), but the later parts of the question proved difficult for many candidates. Many struggled to extract the relevant ingredient from the proof of the Fredholm alternative to show that T is bounded below on a complement of $\ker T$ in (b)(ii)(II). Some candidates put the argument together well in (b)(iii), but others found this challenging. (c)(ii) is a modification of a problem sheet question on complete continuity to work with the weak*-topology, but a number of candidates didn't note that the (f_n) will be uniformly bounded by the principle of uniform boundedness so $(T^* f_n)$ has a convergent subsequence.

C5.1: Solid Mechanics

Q1: This question was attempted by all but one candidates. It was generally well done, though surprisingly few candidates gave an explicit condition for the inversion in part (b)(iii) in terms of the w_i. Very few candidates were able to use the positive definiteness of $\mathbf{B}$ in either part (b)(iv) or (c)(ii).

Q2: This question was fairly popular, but was not particularly well done. Most students tried to use a boundary condition at $r(A) = a$ to determine the unknown constant in part (a)(ii), rather than the observation that $r(0) = 0$ (since material is at the origin and must remain there). Similarly, students did not, in general, realize that $T_{rr} = T_{\theta\theta}$ in this geometry, meaning they were not able to solve the differential equation for T_{rr} required in part (b)(ii). Finally, for part (b)(iv), relatively few students calculated the integral required for the total load, and so did not derive the equation for $\hat{W}$.

Q3: This question was not especially popular, but received relatively high marks on the whole. Part (a) followed lecture material very closely and was done well. Part (b) followed similar lines with a twist; here candidates were generally able to find the sixth order polynomial required for part (b)(ii) but dealing with the different stretches in part (b)(iii) proved more challenging.

C5.2: Elasticity and Plasticity

Question 1 This was the least popular question, but many of those who attempted it managed to get good marks. Some candidates really struggled with the basic geometric identities needed in part (a), but then the non-dimensionalisation and the manipulations needed for parts (b)–(d) were mostly handled well. In part (e), no-one got the point about the curvatures of the beam and the wall matching when $\lambda = 4\pi^2$.

Question 2 This was the most popular question, but had the lowest average mark. The bookwork in parts (a) and (c) was done quite well, although often laboriously and with important steps omitted. In part (b), many students were confused by the fact that $\mathrm{Re}[f]$ was not the same as the stress function used in lectures for a standard Mode III crack problem. In part (d), almost no-one successfully posed and solved the problem in the ζ-plane to determine $f(z)$, although some anyway managed to spot that $f\big(z(\zeta)\big) = -c^2\mathrm{e}^{2\epsilon}/\,(4\zeta^2)$ works. Part (e) was generally fine, although with a lot of minor algebraic slips.

Question 3 This question was a generalisation of a problem sheet question and was reasonably popular, but the average mark was rather low. The solutions were often over-complicated, leading to students getting lost in the algebra. In part (a), several students fallaciously imposed $\tau_{rr} = \tau_{\theta\theta} = 0$ at $r = b$ and then found themselves with too many boundary conditions. In part (b), many students didn't clearly state and apply the correct conditions at the elastic–plastic free boundary $r = s$, and so were unable to close the problem for s. In part (c), few students correctly imposed a purely elastic response on the residual stress to describe the unloading. In part (d), very few students understood that $2P_{c1} < P_{c2}$ is required for the described behaviour to be possible, and almost no-one managed to deduce the given bounds on the parameter β.

C5.5: Perturbation Methods

Overall Question 1 was popular. The first part of the question and the path of steepest descent was generally tackled very well. For the next part of the question, the choice of the appropriate contour for the use of the steepest descent method proved to be a genuine hurdle in the question for a number of candidates, while only the best attempts successfully expanded about the location of the dominant contribution to the steepest descent integral. Many candidates parametrised the steepest descent curve with respect to ζ, without properly accounting for the fact the steepest descent curve has an infinite gradient with respect to ζ at the location of the dominant contribution or recognising that an alternative parametrisation may have been more convenient.

Question 2 appeared to be the least favourite question. A few attempts gathered difficulties early and these candidates generally moved onto the other questions, while there was also a number of very high scoring solutions. Many candidates did not apply the method of intermediate variable matching correctly, while the most successful solutions recognised which terms had to balance when matching via the intermediate variable method.

Question 3 started with bookwork concerning important definitions that candidates knew well. An occasional candidate used a different method than requested in part (b), and a few candidates struggled, but on the whole part (b) was executed very well. The final part differentiated most attempts. In particular keeping track of the level of approximation and the terms that need to cancel between the two integral contributions in the use of the domain splitting method to more than leading order typically, but not always, proved problematic for the candidates.

C5.6: Applied Complex Variables

Question 1

This question was very popular, and there were a lot of good answers. Despite the relative complexity of the setup, all but one candidate identified the correct domains in the potential and hodograph planes (though not all fully justified their figures). Most managed to work through the question successfully. The final equation should have read

$$\phi = \frac{2}{\pi} log|sec 2\alpha|$$

since $sec 2\alpha < 0$. Most candidates did not notice this; those that did correctly assumed that the question was in error.

Question 2 This was a very unpopular question, probably because the set up was slightly less familiar than that in Q1 and Q3. There were only a handful of answers, and very few good ones. Only one candidate realised that the correct approach was to look for a Cauchy integral representation of $w'(z) + \pi i W(z)$. Ironically if instead of asking them to show

$$w'(z) + \pi i W(z) = \frac{1}{2\pi i} \int_\Gamma \frac{d(\zeta)}{\zeta - z} \qquad (\dagger)$$

I had told them that

$$w'(z) + \pi i W(z) = \frac{1}{2\pi i} \int_\Gamma \frac{G(\zeta) d\zeta}{\zeta - z}$$

where $G(\zeta)$ was to be determined then they have found the question much easier. Despite not being able to prove (†) most candidates could use it to solve for w and deduce f.

Question 3 This was a very popular question and there were a lot of good answers. Most candidates successfully identified domains on which $\bar{f}_+$, $\bar{g}_+$ and $\bar{h}_-$. are holomorphic, and the correct values of and α and β.

C5.7: Topics in Fluid Mechanics

Question 1 was straightforward and well done, except for the very last part (the one-dimensional 'phase plane') which baffled everybody.

Question 2 was straightforward and well done until the last part, where drawing the graph of $f(\phi)$ and then $\sum$ was challenging.

Question 3 was straightforward and well done. The ability of candidates to make their way through the algebra of part (c) was encouraging.

In summary, the paper appeared to work well.

C6.1: Numerical Linear Algebra

Q2 was the most popular, and was attempted by over 80% of the candidates. Q1 was attempted by slightly fewer candidates than Q3.

Q1: some struggled to use the Courant-Fischer theorem properly in a(i) to get the desired inequality. a(iv) was a new problem requiring some guessing and computation, and seemed to be very challenging. In (b) some failed to note the assumption $k(A) \gg 1$; when $k(A) = O(1)$ it is easy to come up with examples, but the inequality is not very interesting.

Q2(b): while most correctly used the connection between the power method and QR algorithm to discuss the convergence of the latter, very few noted the requirement in the power method convergence that the initial vector has nonzero components in the the dominant eigenvector. Q2(c)(ii): some presented examples that are triangular or diagonal; while the QR algorithm may not change these matrices much (or at all), this is not a good example as such matrices have already converged! (d)(i) appears to have been very challenging. One needs to use the backward stability of QR factorisation and orthogonal matrix multiplication to prove one step of QR algorithm is backward stable.

Q3(a) (i,ii): A fair number of candidates wrote H^{-1}; this is inappropriate as H is not even square. Some answered (iii)(c) by noting that once the exact solution is found GMRES stops making progress; this is technically correct (and received marks) but the intended solution was to note that GMRES can stagnate even before the solution is found; a fact indicated in a question in the problem sheets. (b) appears to have been challenging, even though it is pretty similar to the discussion in lectures and a question in problem sheets. Most attempts failed to use the orthogonal invariance of Gaussian matrices together with a QR (or SVD) of A.

C6.4: Finite Element Methods for Partial Differential Equations

Q1: This question was attempted by all but one student. It revealed a good spread of abilities across those who attempted it. Q1(a)(i) was answered correctly by every candidate. Q1(a)(ii) was generally answered incorrectly; candidates failed to observe that the basis function ϕ_4 associated with evaluation at the barycentre was zero on the boundary of the cell, and hence did not contribute to the value of the function across a shared edge. Q1(b)(i) was generally answered well, with most candidates invoking the Sobolev embedding theorem as expected; some candidates responded with irrelevant bookwork. In Q1(b)(ii) some candidates did not recall the definition of a direct sum of two vector spaces, or forgot to show that the sum was direct. Q1(b)(iii) and (iv) were answered well by those who attempted them.

Q2: This question was attempted by half of all candidates. This question attracted the stronger students, and was generally answered well. Q2(a) was standard bookwork and was answered well. Q2(b) was again answered well, with marks lost only for minor slips. Q2(c)(ii) challenged some candidates; they did not realise to use the Poincaré inequality on the first term of the right-hand side, and wound up with formulae for ε that were not valid (e.g. requiring $\sqrt{1-K^2}$, where $K > 1$ generally). Q2(c)(iii) was well-answered by those who attempted it.

Q3: This question revealed a good spread of abilities across those who attempted it. Q3(a)(i) was answered correctly by every candidate. In Q3(a)(ii), some candidates failed to justify that $k(x) < M$ for some M; this follows because k is continuous on a compact domain. For the last part of Q3(a)(ii), some candidates invoked characteristic functions to explain why the bilinear form would not be coercive, but such functions are not in H^1; one should instead use bump functions on the subset S of Ω (with nonzero measure) where $k(x) < 0$. Q3(b)(i) was mostly answered well, but every candidate applied integration by parts to the $\nabla \times E$ term to shift the curl operator onto the test function; this is not valid, because the test function for this equation is drawn from $H(\mathrm{div})$, and in general does not have a square-integrable curl. In Q3(b)(ii), candidates sometimes neglected that the question had $\Omega \subset \mathbb{R}^3$, not $\Omega \subset \mathbb{R}^2$, or stated finite elements with scalar-valued spaces instead of vector-valued ones. Q3(b)(iii) was answered well by those who attempted it.

C7.4: Introduction to Quantum Information

Question 1. It was by far the most popular question, attempted by *all* of the students. Perhaps not so surprising, given that the question was based on the mainstream material. The book-work in part (a) was very well answered. In part (b) the students showed a good grasp of the Born rule but many of them struggled with calculations that led to the $\mathrm{Tr}(U)$ expression. Part (c), again, was almost perfectly answered, most likely because the question did not explicitly ask for a detailed calculation of the posterior probabilities. Those few who attempted to calculate the posterior probabilities made minor mistakes, even though they managed to arrive at the right conclusion. Part (d) turned out to be the most difficult one, with many students failing to use the fact that the eigenvalues of a unitary matrix are of the form $e^i\theta$. Instead many attempted to obtain the eigenvalues from the constraints on the trace and the determinant. This is a good alternative approach, but most of the students who took this route could not see the relevance of the *real* part of $\mathrm{Tr}(U)$ when deriving the probability from which the eigenvalues of U are then obtained.

Question 2. In general, the question was well answered and students scored well. Part (a) was book-work but, surprisingly, many students couldn't succinctly justify the answers; part (b) was done in a few different ways, but almost always successfully; in parts (c) and (d) most students dropped a few marks, having struggled with upper bounds; part (e) was usually answered correctly using mathematical induction; part (f) was unproblematic and very few students got it wrong (usually silly mistakes).

Question 3. At first glance this question might have looked difficult for it contained new topics (encryption of quantum states), hence it was not very popular, but those who attempted it did quite well. Part (a) was similar to one of the class problems and most students provided correct answers, but only few supplemented it with geometric interpretation. Students knew how to handle parts (b) and (c) but most did it by analysing specific cases, rather than using general notation. Showing that compositions of Clifford gates are Clifford gates in part (d) posed no problems. Most students noticed that part (e) is a generalisation of part (c) and provided a reasonable description of delegated quantum computation based on Clifford gates. Part (f) was well answered but hardly anyone commented on the need to go beyond the Clifford gates.

C7.5: General Relativity I

Question 1: This question was very popular and attempted by most students. The majority were able to do parts *a-c* without too much difficulty, although a surprising number of students assumed that the curve γ was a geodesic, despite the question explicitly saying that this may not be the case. Those students who struggled with parts *b* and *c* also often seemed to be under the impression that all curves are geodesics. Part *d* required some more algebra, and the ability to convert between abstract tensor expressions and concrete expressions for derivatives of functions along curves – this proved a challenge to a number of students. A frequent error here was believing that the t derivative of the t-component of a vector is always 1, while in fact, in this question, the t-component of the vector in question is a constant (and so its t derivative vanishes). Finally, part *e* should really have been approached as a system of linear ODEs, but almost no students did this. Instead, the majority of students who attempted part *e* derived a second order ODE for one component of Y, and in doing so showed that this component undergoes periodic oscillation – though they rarely went on to show that the other components also oscillate periodically. Overall, most students scored well in the parts of the question they attempted, and low scoring students most often offered partial answers to only a few parts of the questions (the "bookwork" parts) and spent time copying out parts of the question, while leaving other parts of the question completely untouched.

Question 2: This was by far the least popular question, and was only attempted by a handful of students, probably because it was the least familiar in style (compared with past exam questions). Most of the students who did attempt it did well, however, scoring slightly higher on average than the other two questions. Part *a* was not completely straightforward but almost all students were able to do it well, and part *b* required an understanding of normal coordinates and special relativity which was also demonstrated by almost all students. Part *c* was the most difficult part of the question, requiring some fairly intricate algebraic manipulation, and in fact no student was able to completely solve this part of the question, though some came very close. Part *d* was generally done fairly well, even by those students who could not complete part *c*, although no student made explicit the crucial fact that the coordinate vector fields are parallel-transported in Minkowski space.

Question 3: This was a very popular question, with the vast majority of students attempting it together with question 1. Part *a* was done very successfully by almost all students, with only a small minority forgetting that the Lagrangian itself is a conserved quantity (when the curve is parametrised by an affine parameter). Part *b*, however, was generally not done successfully – in fact, no student completely solved this part of the question, though some came very close. A very common error was to assume that a geodesic which is *emitted* radially will always remain radial, whereas in fact (since the spacetime is rotating) the geodesic will itself start to rotate. The key point was to realise that the conserved angular momentum is zero: noticing this made the rest of the algebra considerably easier. Even taking this fact into consideration, the resulting integral was not accurately solved by any student – the easiest way to solve it is to first make a substitution of variables to remove the hyperbolic cosine, and then to remember the formulae for derivatives of inverse trig functions, and while some students were able to perform one of these operations, no student did both. In retrospect this integral is probably too difficult without a hint. Finally, students generally faired better on part *c*, although a surprisingly large number of students made algebraic mistakes in solving the quadratic inequality in part *c (i)* (perhaps they were running out of time when trying this question), and some students made the common mistake of believing that every curve is a geodesic. Finally, most answers to part *c (ii)* were nonsense, and while some students said something true but trivial (e.g. that there are timelike circular orbits only in the interior region – although even this statement is true only if "circular" is interpreted in a coordinate-relative manner), only one student identified the closed timelike curves.

C7.6: General Relativity II

Problem 1 This problem exploring the redshift formula, the stress-energy and the Ricci tensors in the (unnamed) Janis-Newman-Winicour metric was attempted by the majority of the candidates. The typical issues were the following.

- Confusing the coordinate time with the proper time, namely taking the velocity vector of an observer following a curve $\gamma^\mu = (t, r_0, \theta_0, \phi_0)$ with constant r_0, θ_0, ϕ_0 to be simply $(1, 0, 0, 0)$, which is off by a factor of $(g_{tt}|_\gamma)^{-1/2}$.
- Taking the wave vector of a null ray $\gamma^\mu(\lambda)$ to be the velocity vector, as opposed to $\dot{\gamma}^\mu$, where k is the wave number.
- Ignoring or not taking full advantage of the trace reversal in the Einstein field equations.

Judging by the candidates' performance, this problem may have been the most challenging.

Problem 2 This problem on Einstein's quadrupole formula was only tackled by one candidate - and with a very decent level of success. The unpopularity of the problem may indicate the propensity of the students attempting the exam towards more typical problems involving exact metrics.

Problem 3 This problem exploring the Killing horizon of the (unnamed) extremal Kerr solution was attempted by all candidates. The typical mistakes were the following.

- In the context of a hypersurface $\sum$ defined by r = Const, identifying normal vector N with ∂_r instead of taking the normal covector to be $n \propto dr$, as implied by the regular-value theorem.
- Having observed the normal vector to be $N = aT|_\sum + bL|_\sum =: K|_\sum$, where both T and L are Killing vector fields, extending the Killing vector field K away from $\sum$ with non-constant a and b. This reflects the complexity of the concept of Killing horizon, which relies on the non-trivial combination of vector fields defined on and away from it.

C7.7: Random Matrix Theory

Question 1 was attempted by most of the candidates. Parts (a), (b) and (c) were straightforward and were in general answered well. Most candidates found part (d) difficult. Only a few calculated the variance, as asked for; many only established an order estimate for it, but could then still prove almost sure convergence successfully. Only a few candidates correctly identified the paths that give a non-zero contrition in the limit and that the contributions from these can be evaluated using the information given in the question.

Question 2 was attempted by roughly half the candidates. Parts (a), (b) and (d) were straightforward. In answering part (b), some candidates failed to say that the random variables need to be paired with their complex conjugates. Many candidates did well on part (c)(i), but some attempted a more general calculation than was asked for. Part (c)(ii) was challenging and only a few candidates scored well on it. Many didn't see that the permutations fall into two classes, with permutations in each class giving the same contribution. Part (e) was also challenging and only a few candidates scored well on it. Many failed to take advantage of the fact that the matrix entries were stated to be Gaussian random variables and so to use Wick's theorem, despite the question saying to do this.

Question 3 was attempted by roughly half the candidates. Most of those who did attempt it gained high marks. Part (a) was straightforward. Many candidates saw that using the Fourier expansion for the ratio of sine functions considerably simplifies the calculation, but not all did. Most found parts (b) and (c) straightforward too, although some failed to apply Gaudin's Lemma correctly. Several candidates did part (d) well, but some failed to spot the connection to the two-point correlation function, which simplifies the calculation considerably.

C8.1: Stochastic Differential Equations

Question 1. This is a popular question (due to the first question on the paper I believe) attempted by most candidates, while unfortunately there are few good solutions. The first part (a)(i) turns out to be the most challenging part, and very few candidates have idea how to argue independence of Gaussian random variables. By part (a)(i), the part a(ii) should be easy and follows from the martingale property of $M^2 - \langle M \rangle_t$, but still many candidates had no idea how to do it. While most candidates got the marks for a(iii) by using Itô's formula. Candidates find (b) also very challenging, and most of candidates tried to answer this part by using Itô's formula, which is not the direct to do yet though it is possible. While most candidates could not argue properly.

Question 2. This is again a question attempted by most candidates. There are good answers for part (a) which may be answered following step by step (i) - (iv). While candidates had difficult to show the Lipschitz continuity of the coefficients, which is required a bit Prelims analysis. Part (b)(i) is an easy exercise for the exponential martingales, so most candidates got a fair marks, while (b)(ii) seems challenging, some candidates tried to use Itô's formula though the correct way should be the one most elementary: writing down the expectations of both sides in terms of normal distributions.

Question 3. Several candidates attempted this question, but I saw no near complete solutions unfortunately. Parts (a) and (b) are mainly book, including a simple application Lévy's characterisation of Brownian motion. To calculate the pdf in part (c), one should apply Cameron-Martin formula to work out the weak solution, then apply (b). But most candidates who attempted this question just applied Tanaka's formula to the solution directly which leads to a wrong formula.

E. Comments on performance of identifiable individuals

Prizes

Prizes were awarded to the following candidates:

The top prize was awarded to:

Mark Potts (Hertford College)

Prizes were also awarded to:

Maxwell Hutt (New College)
Kai Alexander Bartnick (Somerville College)

The following student was highly commended for his dissertation:

Sebastian Leontica (St Peter's College)

Mitigating Circumstances Notices to Examiners

The Examiners received 16 applications regarding mitigating circumstances. The Examiners considered the applications carefully and agreed appropriate action.

F. Names of members of the Board of Examiners

Examiners:

Prof Alex Schekochihin (Department of Physics, University of Oxford; Chair)
Prof Lionel Mason (Mathematical Institute, University Of Oxford)
Dr David Skinner (Department of Applied Mathematics and Theoretical Physics, University of Cambridge)
Prof Martin Evans (School of Physics and Astronomy, University of Edinbrugh)
Prof Christopher Beem (Mathematical Institute, University Of Oxford)
Prof John Magorrian (Department of Physics, University of Oxford)

Assessors:
Dr Adam Caulton
Dr Abhishodh Prakash
Prof Alan Barr
Prof Aleks Kissinger
Dr Aleksandra Ziolkowska
Prof Alex Ritter
Prof Alex Schekochihin
Dr Alexander Ochirov
Prof Andras Juhasz
Prof Andre Lukas
Dr Andrei Constantin
Prof Andrei Starinets
Prof Andrew Wells
Prof Ard Louis
Prof Aris Karastergiou
Prof Artur Ekert
Dr Attila Szabo
Prof Balasz Szendroi
Prof Bence Kocsis
Prof Caroline Terquem
Christoph Uhlemann
Prof Christopher Beem
Prof Cornelia Drutu
Dr David Alonso
Dr Dylan Butson

Prof Eamonn Gaffney
Prof Fabrizio Caola
Dr Federico Bonetti
Dr Gavin Salam
Prof Gianluca Gregori
Dr Hadleigh Frost
Prof Ian Hewitt
Prof James Binney
Prof James Read
Prof James Sparks
Prof Jason Lotay
Dr Jean-Baptiste Fouvry
Prof John Chalker
Dr John Magorrian
Prof John March-Russell
Prof John Wheater
Prof Jon Chapman
Prof Jon Keating
Prof Jonathan Barrett
Prof Joseph Conlon
Prof Joseph Keir
Prof Julia Yeomans
Dr Lakshya Bhardwaj
Prof Lionel Mason
Dr Lorenzo Tancridi
Prof Lucian Harland-Lang
Prof Louis Fernando Alday
Dr Maximilian McGinley
Dr Melissa van Beekveld
Prof Michael Barnes
Dr Nick Bultinck
Dr Nora Martin
Dr Owen Maroney
Dr Plamen Ivanov
Dr Paul Dellar
Prof Pedro Ferreira
Prof Peter Norreys
Prof Philip Candelas
Dr Prateek Agrawal
Prof Ramin Golestanian
Prof Renaud Lamboitte

Prof Sakura Schafer-Nameki
Dr Sarah Newton
Prof Sid Parameswaran
Prof Simon Saunders
Dr Sounak Biswas
Prof Steve Simon
Prof Steven Balbus
Prof Vlatko Vedral
Prof Xenia de la Ossa
Dr Yizhi You
Prof Yuji Nakatsukasa