**Honour School of Mathematical and Theoretical Physics Part C**
**Master of Science in Mathematical and Theoretical Physics**

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# GEOPHYSICAL FlUID DYNAMICS

## Trinity Term 2023

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**Wednesday, 07th June 2023, 14:30 - 16:30**

*You should submit answers to both questions.*

*You must start a new booklet for each question which you attempt. Indicate on the front sheet the numbers of the questions attempted. A booklet with the front sheet completed must be handed in even if no question has been attempted.*
*The use of a calculator is allowed.*

*The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.*

**Do not turn this page until you are told that you may do so**

1. (a) [5 marks] Using a simple 'parcel exchange' argument in the meridional plane for a stably stratified, baroclinic fluid, briefly outline the conditions under which sloping convection may allow eddies to grow in amplitude. Make sketches of one typical parcel exchange trajectory that releases potential energy and another that doesn't. You should indicate the isopycnals and geopotentials on the sketches.

   (b) [5 marks] A flow with no tilt in the vertical has horizontal velocities $(u, v)$ that can be written as $\Gamma z\, \mathbf{u}_h(x, y)$ where $\Gamma$ is a constant. If the flow is in thermal wind balance, show that it can achieve no horizontal heat transport.

   (c) [9 marks] The Eady model of baroclinic instability predicts a growing normal mode streamfunction perturbation of the form

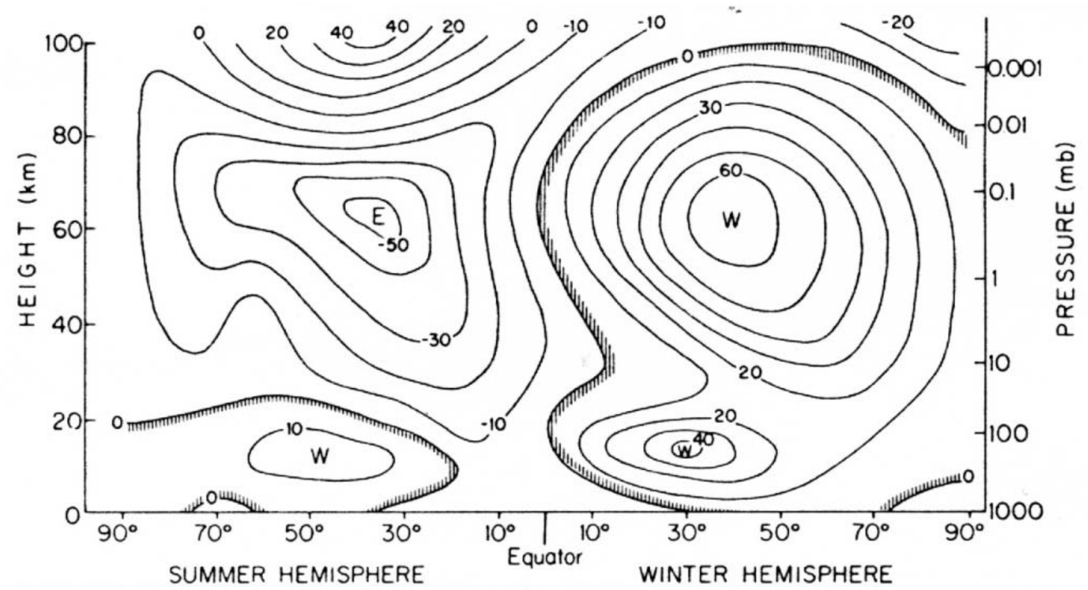
$$\psi' \propto [\cosh(Kz)\cos(kx) - \alpha \sinh(Kz)\sin(kx)]$$

   in the $(x, z)$ plane, where $k$ and $K$ are the zonal and total wavenumbers respectively, $\alpha$ is a positive constant and the domain lies between $z = \pm D$.

   Determine (giving reasoning) whether the streamfunction perturbation tilts to the west or the east with height. Similarly, determine whether the buoyancy perturbation $b'$, which is proportional to $\partial\psi'/\partial z$, tilts to the west or east with height. Explain physically why the perturbation amplitudes are largest on the upper and lower boundaries. Hence sketch the wave structure in the $(x, z)$ plane. (You may assume that the phase difference between the boundaries is a quarter of the zonal wavelength.) Discuss how the vertical tilts facilitate poleward heat transport.

   (d) [6 marks] Why are the atmosphere and ocean required to transport energy poleward? Apart from baroclinic waves, which other circulation structures help to achieve this on Earth, and where are they active?

2. (a) [9 marks] Starting from the equations for geostrophic and hydrostatic balance in pressure co-ordinates, derive the thermal wind equations. The figure below shows the eastward velocity $u$ in $\mathrm{m\,s^{-1}}$ in Earth's atmosphere as a function of latitude and pressure at the solstice. Use this to estimate the temperature drop across the edge of the stratospheric winter polar vortex at around $(60°, 10\text{mb})$. Comment on the magnitude of temperature gradients close to the equator compared to those at higher latitudes, and the extent to which this difference is explained by thermal wind balance.




(b) [5 marks] The quasi-geostrophic potential vorticity equation is given by

$$\left(\frac{\partial}{\partial t} + \mathbf{u_g} \cdot \nabla\right)\left[\nabla_h^2 \psi + f_0 + \beta y + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \psi}{\partial z}\right)\right] = 0,$$

where all symbols have their usual meaning. State the origin of this equation and outline how it can be used as the basis for simple weather predictions.

(c) [11 marks] The linearised Rossby wave equation

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left[\nabla_h^2 \psi' + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \psi'}{\partial z}\right)\right] + \beta\frac{\partial \psi'}{\partial x} = 0$$

can be derived from this by considering streamfunction perturbations $\psi'$ about a uniform background zonal flow $(U, 0, 0)$ with constant buoyancy frequency $N$. Starting from the linearised equation, search for wavelike solutions with $\psi' = A \exp i(kx + ly + mz - \omega t)$ and determine the conditions required for upwards propagation of stationary Rossby waves forced from the surface. Discuss qualitatively where in the figure this is likely to occur. Is the occurrence of upwards propagating stationary Rossby waves affected by which is the Southern and which the Northern Hemisphere?